A BRIEF HISTORY OF CONTROL

By

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In a hole in the ground there began our Larry. That is, Lawrence Markus was born in 1922 at North Hibbing, Minnesota, which was later engulfed by an expanding open-pit iron mine. Shortly thereafter his family moved to Minneapolis, and later to the Chicago area.

Larry entered the University of Chicago, at age sixteen, to study mathematics, physics, and astronomy. He received his B.S. degree in Mathematics after two years, and simultaneously won the University table tennis championship. With America propelled into World War II, Larry was propelled into Meteorology, received his M.Sc., and became an Instructor of Meteorology at the University of Chicago at nineteen. Following a short stint with the Manhattan Atomic Project, he volunteered for the US Navy and served several years aboard a weather frigate in the North Atlantic.

After the war Larry left Chicago for Harvard where he again won the table tennis championship, and completed his Ph.D. in Mathematics following a Fulbright Fellowship in Paris—which he combined with a honeymoon with his wife, Lois. Larry joined the Harvard faculty as an Instructor, then later Instructor at Yale, and Lecturer at Princeton (presaging a Visiting Professorship at Columbia). With Ivy up to his knees, Larry joined the School of Mathematics at the University of Minnesota in 1957, and rose rapidly to become Professor in 1960—and later Regents’ Professor in 1980.

Larry Markus is an unusually erudite and catholic mathematician, with one foot in applied mathematics of dynamics and control theory, one foot in theoretical aspects of geometry and topology, and one foot in abstract algebra. His graduate courses on the geometric algebraic structures of control dynamics were a scandal or an inspiration, depending upon the student. With a lively undergraduate classroom style, he was granted the University Distinguished Teacher Award in 1968.

Larry is a prolific researcher with 130 scientific publications, including the classic treatise “Foundations of Optimal Control Theory” (with E.B. Lee) written while on a Guggenheim Fellowship in Switzerland, and another 15 monographs and editorial collections. His interests were always in seeking the underlying intrinsic algebraic and geometric structures of diverse mathematical phenomena. His Harvard doctoral thesis produced a new way of regarding and analyzing dynamical systems in terms of their topological behavior. In his later work on relativistic cosmology, he concentrated on the global topology

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LAWRENCE MARKUS

LET THERE BE LIGHT!
So went the seminal control command that created the physical universe [Mo] [Ma 6],
and gave it impetus for the explosive expansion of the original Big-Bang [Haw] [Ma 3].
From the viewpoint of control theory the primary question is:

Was the creation an open-loop command initiating the response at a specified
time—say, at \( t = 0 \)?
or

Was this a closed-loop command set to trigger action at a specified state of the
universe—say, at a quantum-mechanical negative-vacuum?

Probably we shall never know the answer since the experiment is too difficult to repeat,
and, despite scientific chutzpah, the state space is too intricate to analyze.

1. Pre-human Control Era.
Within the first few seconds following the Big Bang, the expanding sphere [Ma 2] [Ma 8]
of intense radiant energy became less dense and hot until the radiation began to condense
into protons, electrons and the elementary particles of matter—even into simple hydrogen
atoms (see: modern cosmology mythology). These atoms gravitated into massive gaseous
balls with central pressures and temperatures so great that the hydrogen atoms fused to
produce helium in a nuclear fire. Thus a star is born; and the stage is set for the second
great control experiment.

A star is intrinsically an unstable dynamical system, since the inward gravitational
pull on the surface layers of the star intensifies as the radius decreases, and thus gravity
acts as positive feedback. But the radius of the star can be feedback-stabilized through
the outward radiation pressure generated by the nuclear fusion at the center. If the
gravitational force pulls the surface layers of the star inward from the equilibrium radius,
the central pressure and nuclear fusion intensifies, and then the outward radiation pressure restores the radius towards equilibrium.

But what happens to the star when all the available hydrogen at the center has been converted into helium? Why then the equilibrium fails and the radius collapses, intensifying the central pressure until the helium begins to fuse. This establishes a new stable equilibrium between inward gravitation and outward radiation. But what happens when the helium is exhausted? Why then the succeeding equilibria fail until the star collapses to the point where super-fusions of higher elements occur causing a supernova explosion, fragmenting the star and ejecting the debris of carbon, oxygen, nitrogen, phosphorous, and iron—the elements of life. Thus ultimately a star is a failed control system—unless a supernova instability is deemed a design success.

After this spectacular beginning for the astronomical universe, nothing of importance happens for about 10 billion years (see: current astrophysical mythology). Then commenced the great Gaia experiment. The goddess Gaia (or Gaea) is the Mother Earth who nurtures the biosphere of life, and the human species in particular, (see: classic Greek mythology). The Gaia Hypothesis [G] refers to the action of the biosphere as a feedback stabilizer for the geological dynamics of the Earth’s atmosphere and hydrosphere.

For instance consider the control problem of stabilizing the ratio of the relative densities of CO₂ and O₂ in the atmosphere. If the CO₂ density rises, then the foliage of the tropical rain forests and the plankton of the antarctic oceans luxuriate in the resulting Green House effect. These plants then grow with augmented energy arising from photosynthesis, and thereby consume the CO₂ and release O₂. On the other hand, if O₂ density rises, then forest fires flare and more CO₂ is produced. By such regulator feedback mechanisms this atmospheric equilibrium has been kept quasi-stable for almost a billion years (see: concerned ecological mythology).

Popular misconceptions of these biochemical interactions have led to the belief that “Nature” has a consciousness or intelligent volition, rather than to the recognition of the system-theoretic phenomenon of negative feedback. Will the Gaia Hypothesis and the Biospheric-compensator also turn out to be a failed control system? We do not know as yet, and it seems rather preposterous and presumptuous to predict consequences, or to try to control this complicated system.

2. Artisans and Technicians Control Era.

Since the most antique times, artisans and technicians have skillfully employed control
methods in refining ore, working glass, and most spectacularly in regulating large-scale irrigation processes by the use of water wheels, screw pumps, and dam flues [D] [Nee]. The diorite slab inscribed with the Code of Hammurabi (1700 BC) lists regulatory procedures for communal cooperation in agricultural irrigation [Ham]; and this trend can be followed through the legal Code of Justinian (and the approximately contemporaneous Talmud, about 500 AD), and thence to the Congressional Colorado River Bill of 1956 for reservoir and dam control valid to the year 2000 AD.

It is not entirely clear that such endeavors fit the framework of automatic control, but there are many individual examples of bona fide authenticity. Detailed records over centuries describe the perfection of the float value, for regulating the water level in a tank, by the Greek Ktesibios (250 BC); the development of the water clocks in Egypt and China; and the introduction of an oven temperature regulators by C. Drebbel about 1600 AD [Th].

At the beginning of the industrial revolution, with the great advances in instrumentation, we find pendulum clocks improved by Christian Huygens and reaching perfection in the chronometer of John Harrison (1763), with an intricate design of motion and temperature compensators. But certainly Watt’s application of the centrifugal governor for regulating steam engines (1775) can be taken as the start of modern control engineering.

3. Early scientific Control Era (1850–1900).

In 1868 J. C. Maxwell published a mathematical analysis of Watt’s governor and related mechanical regulators. In his famous paper “On Governors”, Maxwell [Max] applied the principles of Newtonian mechanics to describe the action of such a governor by linear differential equations with constant coefficients. He recognized that a necessary and sufficient condition for (asymptotic) stability was that the corresponding characteristic polynomial should have roots only in the left-half complex plane. After resolving the stability question for third degree equations, he then declared: “I have not been able completely to determine these conditions for equations of a higher degree than the third; but I hope that the subject will obtain the attention of mathematicians”.

For real polynomials the stability problem was attacked by C. Hermite and then totally resolved in a famous paper of A. Hurwitz (1894) [Hu]. The applications to engineering were emphasized by J. Vishnegastrakii (1876) [V].

Yet here we are, over a century later, still at it—for partial differential equations and functional differential equations—with Hardy space $H^\infty$, Paley-Wiener theory, and matri-
ces over Noetherian rings! (see: Abstract mathematical mythology).

The research in dynamics, especially stability theory including the work of Routh (1877), is well reported in the foundational two volumes “Theory of Sound” by Baron Rayleigh (1894) [Ra]. It is interesting to note that in the section on power line transmission in volume I, Rayleigh quotes the earlier work of Heaviside “... circuit ... is wholly cleared of electrification and current in the time t/v”. This was in connection with feedback boundary damping for the one-dimensional wave equation by what is now called “matched impedance”.

The other great treatise of this era on the stability of dynamical systems was the remarkable volume “Problème général de la stabilité du mouvement” by A. Lyapunov (1892) [Ly1] [Not to be confused with the important paper on the mathematical basis of bang-bang control “Sur les fonctions-vecteurs complètement additives” by a different A. Lyapunov (1940) [Ly2]]. Here Lyapunov (the first one) introduced two methods of analyzing the stability of a vector nonlinear differential system: Method I consists of linearization and relates to the local behavior near an equilibrium; Method II depends on dissipation of an energy (Lyapunov) function and yields a global result. These two methods of stability analysis lie at the heart of many subsequent investigations over the past century.


The central problem of control engineering for the first half of this century was the accurate reproduction and amplification of a signal on a telephone cable. This amplification, only practical following De Forest’s invention of the triode tube, was essential for long distance communication, and the problems posed and resolved led to the classical methods of control engineering.

In the opening decades of the century O. Heaviside and C. Steinmetz pioneered the path through the algebraization of linear differential systems by means of operator calculus and complex analysis. Although these methods had been known in a mathematical context since the times of Lagrange and Laplace, they were now available in a format that matched the approaches of electrical engineers. But the most important discoveries were by H. Nyquist ("Regeneration Theory", 1932) [Ny], and shortly thereafter, H. Bode and W. Evans. These papers first introduced the transfer function for a feedback circuit, and then developed practical graphical methods for analyzing the stability of the circuit for various possible compensators. Essentially the Nyquist plot, the Bode diagram,
and the Evans root-locus technique all depend on classical results in complex function theory—particularly the Principle of the Argument.

In retrospect the fundamental paper of Nyquist appears unnecessarily complicated. Instead of defining the transfer function around a feedback loop in terms of an equilibrium condition, it is constructed by considering an infinite series of regenerations of the current around the loop. The complex analysis repeats much of the classical analysis of complex integration theory with hypotheses of integrands of “bounded variation”, and integration paths “over the appropriate Riemann surface”. This exposition caused the importance of the basic discovery to be misjudged by many electrical engineers, who laughed at the paper as a “snow job” [Th]. Nevertheless Nyquist does come to his following:

“Rule: Plot plus and minus the imaginary part of $A J(iw)$ against the real part for all frequencies from 0 to $\infty$. If the point $1 + iw$ lies completely outside this curve the system is stable; if not it is unstable”.

While much of this research was accomplished at Bell Telephone Laboratories, and later at the MIT Radiation Laboratory [JNP], in connection with problems of telephone and radar operation, there is another work of that era which is little known and which has had great consequences, as explained to me by my colleague, Alfred Nier, Regents’ Professor of Physics at the University of Minnesota. In 1935 Nier published a short note on the operation of the mass spectrometer [Ni], in which he describes the problem:

“In practically all methods of positive ray analysis a combination of an electric and a magnetic field is used to separate the ions of different masses . . . The fact that the deflection suffered by an ion depends upon both the magnetic field and the electric field suggests that instead of attempting to keep the magnetic field constant one could cause the electric field to fluctuate automatically along with the magnetic field in such a manner that the deflection of an ion would be independent of the magnetic field fluctuations.

“This can be accomplished very simply by a vacuum tube amplifier.”

This enormously important discovery by Nier made the mass spectrometer a practical instrument for guiding the separation of Uranium 235 and 238. It was the key to all future development in the production of atomic energy, and research in nuclear physics.

In summary the classical control era was concerned with the stabilizing of feedback compensating circuits, which were described mathematically in terms of linear ordinary differential equations with constant coefficients. A few attempts had been made to extend
these methods to equations with time-varying coefficients, as with the describing-function of Kochenburger [Ko], and to allow for some stochastic noise and signal delays [Mi2], but such developments were largely left to future generations.

5. Innovations of Control in WWII Era.

During the Second World War the main stimulus to control engineering arose from the development of new technologies of weaponry, especially in aeronautics and marine navigation. These constructions, of jet fighter and long-range bomber aircraft and ballistic missiles in military offense, as well as radar and electronic detection and navigation in defense, all demanded high level innovations of science and engineering.

In the USA much of this research was conducted at MIT, particularly in the Radiation Laboratory that was mainly concerned with the perfection of radar. The control problem was to use the radar data as feedback to guide the gun tracking of enemy aircraft. Thus there were major efforts in the control theory of target-tracking and also target-prediction. New sophisticated mathematics theories involving stochastic processes and statistical methods of functional analysis came to prominence, led by the researches of Norbert Wiener [W]. Some of these topics, specially the more applicable results, were reported in the influential volume "Theory of Servomechanisms" by James, Nichols and Phillips [JNP].

In the USSR the theory of nonlinear vibrations and oscillations was advanced by Krylov and Bogoliubov [Bo K], and their followers, while strong cohorts of theoretical engineers were led by A. M. Letov [Let], A. I. Luré and many others. The prediction theory of Kolmogorov was developed independently from that of Wiener.

Any detailed study of the control advances in the era would require volumes to record, and more scholarship than I possess. Essentially this would constitute a great chapter in the history of technology, and so I leave this topic with only the few remarks offered above.


It is always easier to write a survey about something on which your knowledge is restricted to vague generalities, rather than to report accurately on a complex collection of events in which you are an individual participant. In this latter case the report is colored by your own attitudes and aptitudes. Accordingly, this brief sketch of control science of the decades following 1950 must be regarded as a personal history, rather than an objective chronicle.
Shortly after the war Solomon Lefschetz, a world famous topologist and past President of the AMS, argued that America should have a national policy of potent strength in fields of applied mathematics. With the enthusiastic encouragement of the Office of Naval Research, Lefschetz established a center for research in nonlinear ordinary differential equations (ODE) and dynamics at Princeton University—somewhat in parallel to the new Courant Institute at NYU which specialized in partial differential equations (PDE). Lefschetz was mainly interested in nonlinear oscillations and stability theory, as evidenced by his text [Lef]. After Lefschetz retired from Princeton, he moved his research group first to RIAS (research institute of Martin Corporation) and later to Brown University where it became the Lefschetz Center for Dynamical Systems under the direction of Joseph La Salle, with the cooperation of Wendell Fleming, E. (Jim) Infante, Harold Kushner, and others.

I joined the Princeton group as Lecturer and Lefschetz' assistant (1955–1957), and helped to organize his weekly seminar. There were many senior mathematicians who participated in these programs including R. Bellman, L. Cesari, J. La Salle, J. Leray, L. S. Pontryagin, as well as more junior experts like A. Antosiewicz, R. Bass, S. Diliberto, J. Hale, R. Kalman, J. Moser, M. Peixoto, G. Reeb, and S. Smale.

Lefschetz who was then a quite elderly mathematician (but no older than I am now [Ma 5]), was born in Moscow in 1884 but began his professional career as an engineering graduate of the Ecole Centrale of Paris [Ma 7]. Later he emigrated to the United States as an industrial engineer. But in a tragic engineering accident he lost both his arms, and thereafter used artificial limbs and hands with great skill in daily life and even in his manifold writings and blackboard-chalk lectures. His discoveries in algebraic topology and algebraic geometry were among the high points of twentieth century mathematics. His enthusiasms and scientific insights were an inspiration to his many students and colleagues and he was certainly a major influence on my mathematical development.

Many of the mathematicians and theoretical engineers from the Princeton-RIAS-Brown groups later established their own centers of research. At the University of Minnesota I cooperated with my colleague Bruce Lee in organizing the Center for Control Science and Dynamical Systems. I was the first Director of this Center, and later Bruce took over that responsibility with the help of other colleagues like K. S. P. (Pat) Kumar. Our scientific cooperation culminated in the writing of our joint treatise, Foundations of Optimal Control Theory (1967) [LM], which was widely used and subsequently translated into Russian and into Japanese.
In 1968 I became associated with the University of Warwick, England, where I alternated my duties with Minnesota. At Warwick, with the encouragement of E. C. Zeeman, I established and directed the Control Theory Centre (later directed by P. C. Parks and A. J. Pritchard).

The lists of lecturers and visitors at these two Centers (Centres) incorporate many of the most distinguished control scientists and world leaders in related mathematical and engineering fields. My own research activities, and those of my doctoral students, emphasized nonlinear ordinary differential equations—especially the geometric and algebraic aspects of control theory, (lately more in the direction of PDE and hybrid control systems, see [Li M1], [Li M2], and [MY]). Bruce Lee became a world authority in delay-differential (or more general hereditary functional differential equations (FDE)) control systems, with an entire school of doctoral and post-doctoral students following his leadership.

Meanwhile, back in the USSR at the Steklov Institute in Moscow, L. S. Pontryagin had established a somewhat similar concentration of young mathematicians with strong interests in nonlinear differential equations. These included his fellow Lenin Prize winners, V. Boltyanski, R. Gamkrelidze, and E. Mischenko, who coauthored the famous seminal book, “The mathematical theory of optimal processes”[PBGM]. Other younger associates, now famous mathematicians, included D. Anosov and V. I. Arnol’d.

Like Lefschetz, Pontryagin was world famous as an algebraic topologist (homotopy groups, cohomology classes) and as the creator of the duality theory for topological groups. He, too, had suffered an early accident which left him blinded. Also, although it was not widely known by the mathematical establishment, Pontryagin had long pursued a creative career in applied mathematics. For instance, Pontryagin generalized the Hurwitz stability criterion to cover linear delay-differential equations [P1], which have exponential-polynomial characteristic functions. Also it was he who first introduced the concept of structural stability (robustness) for dynamical systems [P2].

Other important research centers in the USSR developed around Mitropolsky in Kiev, Krasovskii and Mishkins in Sverdlovsk, as well as other powerful scientific institutions in Leningrad and Moscow. The famous Institute of Automatic Control and Telemechanics in Moscow was directed by A. I. Lu˘re and later by A. Letov (and A. Ya. Lerner) for several years, after which Letov became the director of the center for environmental control at the International Institute for Applied Systems Analysis (IIASA) near Vienna.

The build-up of research in control theory proceeded at the Universities of Oxford, Cambridge, Warwick, Manchester, and Imperial College in London. In France the Paris
based IRIA was led by J. L. Lions, and other institutes soon followed in the rest of Europe as well as in Japan and in South America. It would be futile, and also presumptuous, for me to try to give any full impression of the manifold of distinguished research centers and institutes that had established scientific centers of cooperation between mathematicians and control engineers by the 1970’s. My personal interactions were with A. V. Balakrishnan [Ba] and L. Neustadt in California.

As this era came to full flowering, it could be asserted that, from an overall viewpoint, the mathematical researches in the areas of control science were expanding into new areas:

(i) Distributed parameter systems (PDE) based on the revolutionary treatises of A. G. Butkovskiy [Bu] and J. L. Lions [Li 1]. The former staying closer to the engineering tradition, and the latter bringing into play the full Bourbaki apparatus of modern functional analysis and partial differential equations.

(ii) Nonlinear differential systems (ODE) involving new methods of Lie algebras and Lie groups—both in the deterministic and the stochastic cases—with generalizations stretching towards (FDE).

(iii) Linear systems theory founded on the concepts of controllability, observability, and identification. Here the interplay between the time-domain and the frequency-domain was to be exploited by a virtuosity of abstract algebra, with the lead taken by Rudy Kalman, and H. Rosenbrock.

7. An anecdotal history of two astonishing surprises.

At the International Congress of Mathematicians in Edinburgh in 1958 L. S. Pontryagin presented a major invited address “Optimal Processes of Regulation” [P3]. At that time Russians in the West were exotic phenomena, and the excitement of the occasion was inflated by the fame and mathematical eminence of Pontryagin—the great topologist who was renowned for his researches into cohomology by means of characteristic classes and for the duality theory of topological groups. The lecture hall was overflowing and Lipman Bers had assumed his place for an English simultaneous translation of the lecture.

The international mathematical audience, led by a concentration of abstract topologists, were flabbergasted and astonished as the lecture developed. Pontryagin seemed to be talking about some kind of engineering, leaving them feeling ignorant and confused. The Maximum Principle of Pontryagin seemed mysterious and incomprehensible, except to those relatively few who were experts on control theory and who were already familiar with the Bang-Bang principle in the linear case treated previously by Bellman [BGG]. I
recognized this direction of mathematical analysis, and also I had previously met Pontryagin (as Lefschetz’ assistant) and knew very well of his long-standing interest in the problems of applied dynamical systems.

After a few days of reflection and gossip at the Congress, the mathematical establishment decided that the Maximum Principle wasn’t about engineering, after all, but was instead a topic in the classical calculus of variations (similar to earlier results of M. Hestenes [Hes]). Thus the consensus of the Congress came to the conclusion that control theory might be mathematically respectable, but that it was dull and boring—the position still held by many of the avant-garde of abstract mathematicians.

In 1961 R. Kalman and R. Bucy published their remarkable paper [BuK] on the optimal control of linear systems, with Gaussian disturbances, and relative to a quadratic cost functional. The mathematical result was that the optimal controller could be synthesized through the solution of a Riccati matrix differential equation. Accordingly, a reasonably elementary computer program could generate the desired optimal controller, which also served as a feedback stabilizer.

Not only was this analysis elegant mathematics, but more importantly, it led directly to practical hardware for engineering control of guidance devices. In particular, the Kalman-Bucy filter was an absolutely essential component of the guidance system for the Apollo Moon-Rocket.

The industrial engineering community were flabbergasted and astonished that any such “fancy mathematics” could lead to useful and important engineering products. Thus mathematical control theory was gradually accepted and deemed to be potentially useful and practical. For many years, whenever control theoreticians were challenged to defend their craft to corporate engineering directors or to governmental funding agencies, the automatic and parrot-like response would ring out, “Kalman filter”.


During the past quarter century new trends have appeared and flourished in the mathematical theory of control systems. Some of the recent branches on the tree of mathematical control are:

i) Algebraization of frequency-domain analysis, leading into the realm of ring theory and algebraic geometry. The works of R. Kalman [Kal] and his colleagues, particularly E. Sontag, are fundamental developments.

ii) Partial differential equations and functional analysis for distributed parameter
systems, and systems with infinite dimensional state spaces. Pioneers in this field include H. Fattorini [Fa], J. L. Lions [Li2], D. Russell [Rus]. New researches are carried forward by their students and colleagues.

iii) Lie groups and algebras in a geometric treatment of nonlinear control systems. See papers of R. Brockett [Br], H. Hermes [HL], A. Isidori [Is], H. Sussmann [JS], et al. Also I should recall the historic role played by C. Carathéordory (1909) in his axiomation of classical thermodynamics, where the reachable states under cyclically reversible processes are described by Lie algebra techniques [Ca].

iv) Functional differential equations, especially hereditary or delay-differential equations, are an active field for investigations by J. Hale and E. B. Lee, each with schools of doctoral students.

v) Frequency-domain analysis for infinite dimensional systems, as described by PDE or FDE, leading to new types of problems in complex analysis. In particular, Hardy space $H^\infty$ is at the center of these studies. This is paradoxical and is much to the chagrin of Hardy’s spirit, in view of the total scorn and hostility to applications frequently expressed in the writings of Hardy [Har]

vi) Fields so new and under such continual turbulent re-evaluation (e.g. robotics, computer logic, and bioengineering based on physiological homeostasis) that their influence on control theory or vice versa is impossible to assess at this time.

But for insights into the future research for the next quarter century, one could do no better than to review the contributions to this Festschrift volume.

While all these developments of control theory may have been of only uncertain value to control engineering, there is no question but that they have been a bonanza for mathematics.

January 1993

Acknowledgement.

I thank the editors for inviting me to prepare this brief essay on the development of mathematical control theory, and to record some personal reminiscences, as suggested in the Preface.
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