A NOTE ON SOLUTIONS FOR
THE INTRINSIC GENERALIZED WAVE
AND SINE-GORDON EQUATIONS

By

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IMA Preprint Series # 575
August 1989
A NOTE ON SOLUTIONS FOR THE INTRINSIC GENERALIZED WAVE AND SINE-GORDON EQUATIONS

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0. Introduction

In the classical theory of differential geometry, the sine-Gordon equation was associated to surfaces of constant negative curvature contained in the euclidean space $\mathbb{R}^3$. A generic solution of this equation represents the angle between the asymptotic curves of the surface. In 1875, Backlund studied the geometry of these surfaces and obtained a transformation which provides new solutions for the sine-Gordon equation from a given one.

A generalization of these results led to a generalized sine-Gordon equation [9][10] and a generalized wave equation [8] and their Backlund transformations. Moreover, it was shown in [1][2] that the inverse scattering method can be applied in order to obtain solutions for both equations. Solutions for the generalized sine-Gordon and wave equations are orthogonal matrix functions of $n$ independent variables, which correspond respectively to hyperbolic $n$-dimensional submanifolds of the euclidean space $\mathbb{R}^{2n-1}$ and flat submanifolds of the unit sphere $S^{2n-1}$.

Another geometric interpretation for the classical sine-Gordon equation and other evolution equations such as KdV, MKdV, Burgers and many others was given in [4][5][6][7], where these equations were shown to be associated to the following intrinsic problem: generic solutions of such equations define hyperbolic metrics on open subsets of $\mathbb{R}^2$.

Motivated by the two-dimensional case, an intrinsic generalization for the sine-Gordon and wave equations was introduced in [3]. Moreover, a transformation was obtained which provides new solutions from a given one and it was also shown that

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*Research partially supported by the Institute for Mathematics and its Applications with funds provided by the NSF and CNPq, Brazil.
solutions for these equations can be obtained by applying the inverse scattering method. Solutions for these equations are unit vector fields in $\mathbb{R}^n$, which define metrics with constant curvature on open subsets of $\mathbb{R}^n$.

In this note, taking $n = 3$, we consider the unit vector fields determined by two angle functions and rewrite the intrinsic generalized sine-Gordon and wave equations in terms of these functions. Moreover, using the results given in [3], we obtain explicit 1-soliton solutions and provide a graph of the angle functions.

1. The intrinsic generalized wave and sine-Gordon equations

The intrinsic generalization for the wave and sine-Gordon equations was introduced in [3] and it was motivated by a geometric problem, related to Riemannian manifolds of constant curvature.

We consider pairs $\{v,h\}$ of functions in the variables $x_1,\ldots,x_n$, where $v$ is a unit vector field and $h$ is an off diagonal matrix function defined on an open subset of $\mathbb{R}^n$, which satisfy the following set of equations:

\begin{align}
vv^t &= 1 \quad (1) \\
\frac{\partial v_i}{\partial x_j} &= v_j h_{ji} - \sum_{s=1}^{n} v_i h_{is} \delta_{ji} \quad (2) \\
\frac{\partial h_{ij}}{\partial x_i} + \frac{\partial h_{ij}}{\partial x_j} + \sum_{s \neq i, s \neq j} h_{is} h_{sj} &= -k v_i v_j, \quad i \neq j, \quad (3) \\
\frac{\partial h_{ij}}{\partial x_i} &= h_{is} h_{sj}, \quad i, s, j \text{ distinct} \quad (4) \\
\frac{\partial h_{ij}}{\partial x_i} + \frac{\partial h_{ij}}{\partial x_j} + \sum_{s \neq i, s \neq j} h_{js} h_{is} &= 0 \quad i \neq j. \quad (5)
\end{align}

The set of equations (1)-(5) is called the Intrinsic Generalized Wave Equation (IGWE) when the constant $k = 0$ and the Intrinsic Generalized Sine-Gordon Equation (IGSGE) when $k = -1$.

Remark: Whenever the coordinate functions of the vector field $v$ do not vanish, the off diagonal matrix $h$ is determined by $v$. Moreover, (5) is a consequence of the previous ones. The full set of equations is considered, in order to allow solutions which do not satisfy the above condition.
Given a solution of (1)-(5), such that the coordinate functions \( v_i \) do not vanish on an open subset \( U \) of \( \mathbb{R}^n \), we can define a metric on \( U \) by \( \langle \frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_j} \rangle = \delta_{i,j} v_i^2 \), for which the sectional curvature is constant equal to \( k \).

Observe that when \( n = 2 \), the above equations reduce to the homogeneous wave equation and the sine-Gordon equation respectively. In fact, consider the unit vector field in \( \mathbb{R}^2 \)

\[
v(x_1, x_2) = \left( \cos \frac{u}{2}, \sin \frac{u}{2} \right),
\]

where \( u(x_1, x_2) \) is a differentiable function. Then the matrix \( h \) is given by

\[
h = \frac{1}{2} \begin{pmatrix}
0 & u_{x_1} \\
-u_{x_2} & 0
\end{pmatrix},
\]

(3) reduces to

\[
u_{x_1 x_1} - u_{x_2 x_2} = -k \sin u.
\]

and (4),(5) are trivially satisfied. We also observe that the intrinsic generalized equations are nonlinear for \( n > 2 \), even when \( k = 0 \).

In general, for any dimension \( n \), one can consider the unit vector field \( v \) given by \( n - 1 \) functions of the variables \( x_1, \ldots, x_n \). Then (1)-(5) reduce to a system of differential equations for these functions. In what follows, this will be done for \( n = 3 \).

We consider

\[
v = (\sin \varphi \sin \theta, \sin \varphi \cos \theta, \cos \varphi), \tag{6}
\]

where \( \varphi \) and \( \theta \) are functions of \( x_1, x_2, x_3 \). It follows from (2) that the off-diagonal matrix \( h \) in terms of \( \varphi \) and \( \theta \) is given by

\[
h = \begin{pmatrix}
0 & \cotg \varphi \cotg \theta \varphi_{x_1} - \theta_{x_1} & -\varphi_{x_1} / \sin \theta \\
\tan \theta \cotg \varphi \theta_{x_2} + \theta_{x_2} & 0 & -\varphi_{x_2} / \cos \theta \\
\sin \theta \varphi_{x_3} + \tan \varphi \cos \theta \theta_{x_3} & \cos \theta \varphi_{x_3} - \tan \varphi \sin \theta \theta_{x_3} & 0
\end{pmatrix}.
\]

Equation (3) is symmetric with respect to \( i, j \), therefore we only need to consider \( i < j \). Hence, (3) is given by the following three equations.

\[
\begin{align*}
\varphi_{x_1}^2 &\sin^2 \varphi \sin^3 \theta \cos^2 \varphi \cos^3 \theta + \varphi_{x_1} \theta_{x_2} \sin^3 \varphi \sin^2 \theta \cos \varphi \cos^2 \theta (\cos^2 \theta - \sin^2 \theta) \\
+ \varphi_{x_2} &\sin \varphi \sin^5 \theta \cos \varphi \cos \theta - \varphi_{x_2}^2 \sin \varphi \cos^3 \theta + \varphi_{x_2} \theta_{x_2} \sin \varphi \sin^2 \theta \cos^3 \varphi \\
+ \varphi_{x_1} \sin \varphi \cos^3 \theta - \varphi_{x_1}^2 \sin \varphi \cos^2 \varphi \cos^3 \theta - \varphi_{x_1} \theta_{x_1} \sin \varphi \cos^3 \varphi \cos^2 \theta - \theta_{x_1}^2 \sin^4 \varphi \sin^3 \theta \cos \varphi \cos^2 \theta \\
- \theta_{x_1} &\sin \varphi \sin^2 \theta \cos^2 \varphi \cos^2 \theta = -k \sin^4 \varphi \sin^3 \theta \cos^2 \varphi \cos^3 \theta
\end{align*}
\]

(7)
\[
\varphi_{zzz_1} \sin \varphi \cos^3 \varphi \cos^2 \theta + \varphi_{zz} \theta_{zz} \sin \varphi \sin^2 \theta \cos^3 \theta (1 + \cos^2 \varphi)
- \varphi^2_{z_1} \sin^3 \theta \cos^3 \varphi - \varphi_{z_2} \theta_{z_2} \sin \varphi \sin^2 \theta \cos^2 \varphi \cos \theta - \varphi_{z_1} \theta_{z_1} \sin \varphi \sin \theta \cos^2 \varphi \cos^2 \theta + \varphi_{z_1} \theta_{z_1} \sin \varphi \cos^2 \varphi \cos^2 \theta + \theta_{zzz_1} \sin^2 \varphi \sin^2 \theta \cos \varphi \cos^3 \theta
- \theta^2_{z_1} \sin^3 \theta \cos \varphi \cos^2 \theta = -k \sin^2 \varphi \sin^3 \theta \cos^3 \varphi \cos^2 \theta
\]  

(8)

\[
\varphi_{zzz_2} \sin \varphi \sin^2 \theta \cos^2 \varphi \cos^3 \theta - \varphi_{zz} \theta_{zz} \sin \varphi \sin^3 \theta \cos^2 \theta (1 + \cos^2 \varphi)
- \varphi_{z_2} \varphi_{z_2} \sin \varphi \sin^2 \theta \cos \varphi \cos \theta - \varphi_{z_2} \theta_{z_2} \sin \varphi \sin^3 \theta \cos^2 \varphi - \varphi_{z_1} \varphi_{z_1} \sin \varphi \sin \theta \cos \varphi \cos^2 \theta + \theta_{zzz_2} \sin^2 \varphi \sin^2 \theta \cos \varphi \cos^3 \theta
- \theta^2_{z_2} \sin^2 \varphi \sin^2 \theta \cos \varphi \cos^3 \theta = -k \sin^2 \varphi \sin^2 \theta \cos^3 \varphi \cos^2 \theta
\]  

(9)

(4) provides six equations, which reduce to the following:

\[
\varphi_{zzz_1} \sin \varphi \sin \theta \cos^2 \varphi \cos \theta - \varphi_{zz} \varphi_{z_1} \sin \theta \cos^3 \varphi \cos \theta
- \varphi_{z_1} \theta_{z_1} \sin \varphi (1 - \sin^2 \varphi \cos^2 \theta) - \theta_{zzz_1} \sin^2 \varphi \sin^2 \theta \cos \varphi = 0
\]  

(10)

\[
\varphi_{zzz_2} \sin \varphi \sin \theta \cos^2 \varphi \cos \theta - \varphi_{zz} \varphi_{z_2} \sin \theta \cos^3 \varphi \cos \theta
+ \varphi_{z_2} \theta_{z_2} \sin \varphi (1 - \sin^2 \varphi \sin^2 \theta) + \theta_{zzz_2} \sin^2 \varphi \cos \varphi \cos^2 \theta = 0
\]  

(11)

\[
\varphi_{zzz_1} \sin \varphi \sin \theta \cos \theta - \varphi_{zz} \varphi_{z_1} \sin \theta \cos \varphi \cos \theta
+ \varphi_{z_1} \theta_{z_1} \sin \varphi \sin^2 \theta - \varphi_{z_1} \theta_{z_1} \sin \varphi \cos^2 \theta = 0
\]  

(12)

Equation (5) is trivially satisfied.

We conclude that for \( v \) given as in (6), the intrinsic generalized equations reduce to a system of second order differential equations (7)-(12) for two functions \( \varphi, \theta \) depending on \( z_1, z_2, z_3 \).

We observe that taking \( \varphi = \pi/2 \) and \( \theta \) depending on two variables, \( \theta(x_1, x_2) \), then the system of equations reduce to the classical wave or sine-Gordon equations. In fact equation (7) reduces to

\[
\theta_{z_1 z_1} - \theta_{z_2 z_2} = -k \sin \theta \cos \theta,
\]

and the others are trivially satisfied.
2. Solutions for the IGWE and IGSGE.

In this section we recall the transformations obtained in [3], which provide new solutions for the IGWE and IGSGE from a given one. Considering the three dimensional case, \( n = 3 \), we obtain 1-soliton solutions for the equations and the corresponding functions \( \varphi \) and \( \theta \) that were introduced in the previous section.

In order to state the results, we rewrite (1)-(5) in matrix notation. Let \( e_j \) be the \((n \times n)\)-matrix given by \((e_j)_{k \ell} = \delta_{kj}\delta_{ij}\), and \( h \) an off-diagonal matrix. We define the matrix 1-forms

\[
E = \sum_{j=1}^{n} e_j dx_j \\
B = -Eh + h^t E \\
C = hE - Eh^t
\]

Using this notation in (1)-(5), the IGWE is given by

\[
vv^t = 1 \\
dv = -vB \\
dC = C \wedge C \\
dB = B \wedge B,
\]

and the IGSGE is written as

\[
vv^t = 1 \\
dv = -vB \\
dC = C \wedge C + \frac{1}{2} E \wedge QE \\
dB = B \wedge B,
\]

where

\[
Q = 2v^t v - I
\]

and \( I \) denotes the \( n \times n \) identity matrix.

In the following results \( Y^0 \) denotes the off-diagonal part of the matrix \( Y \).

Theorem 1. Suppose \( \Omega \) is a simply connected domain in \( \mathbb{R}^n \) and \( \{v, h\} \) is a solution of the IGWE defined on \( \Omega \). Then for each \( z \in \mathbb{R} \), the system of equations

\[
dY = z(E - YEY) - YC + BY
\]
has a unique solution \( Y : \Omega \rightarrow M_n(\mathbb{R}) \) having a prescribed value at a given point of \( \Omega \). If this prescribed value is an orthogonal matrix, then the solution \( Y \) is in \( O(n) \). Moreover, \( \{\tilde{v}, \tilde{h}\} \) defined by

\[
\tilde{v} = vY \\
\tilde{h} = h^t - zY^0
\]  

(18)

is a new solution for the IGWE.

The analogous result for the IGSGE is given by

**Theorem 2.** Suppose \( \Omega \) is a simply connected domain in \( \mathbb{R}^n \) and \( \{v, h\} \) is a solution of the IGSGE defined on \( \Omega \). Then for each \( z \in \mathbb{R} \setminus 0 \), the system of equations

\[
dY = \frac{z}{2}(E - YEY) + \frac{1}{2z}(QE - YEQY) - YC + BY
\]  

(19)

has a unique matrix valued solution \( Y : \Omega \rightarrow M_n(\mathbb{R}) \), having a prescribed value at a given point of \( \Omega \). If this prescribed value is in \( O(n) \), then the solution \( Y \) is in \( O(n) \). Moreover, \( \{\tilde{v}, \tilde{h}\} \) defined by

\[
\tilde{v} = vY \\
\tilde{h} = h^t - \frac{z}{2}Y^0 - \frac{1}{2z}(QY)^0
\]  

(20)

is a new solution for the IGSGE.

A proof of Theorems 1 and 2 can be found in [3]. Now we restrict ourselves to \( n = 3 \) and we apply the above results to obtain explicit solutions for the IGWE and IGSGE, by starting with the trivial solution \( v = (1, 0, 0) \) and \( h \equiv 0 \) and hence \( B = C = 0 \).

From Theorem 1 we want to solve

\[
dY = z(E - YEY)
\]  

(21)

for a given prescribed orthogonal \( Y \) at the origin. So we consider

\[
Y(0) = \frac{1}{a^2 + b^2 + c^2 + 1} \begin{pmatrix}
    a^2 + b^2 - c^2 - 1 & 2(a + bc) & 2(-ac + b) \\
    2(a - bc) & -a^2 + b^2 - c^2 + 1 & -2(ab + c) \\
    -2(ac + b) & 2(ab - c) & -a^2 + b^2 + c^2 - 1
\end{pmatrix}
\]  

(22)
where \( a, b, c \) are real constants. Then it is not difficult to see that the unique solution \( Y(x_1, x_2, x_3) \) of (21) with this initial condition is given by

\[
Y = \frac{1}{D} \begin{pmatrix}
\sigma_{12} b^2 - \sigma_{13} c^2 - \sigma_{12}^2 + a^2 & \sigma_{12} (e^{2x_{23}} bc + a) & 2\sigma_{13} (e^{2x_{23}} b - ac)
\
\sigma_{12} (e^{2x_{23}} bc + a) & \sigma_{23} b^2 - \sigma_{13} c^2 + \sigma_{12}^2 - a^2 & -2\sigma_{23} (e^{2x_{1}} c + ab)
\
-2\sigma_{13} (e^{2x_{23}} b + ac) & -2\sigma_{23} (e^{2x_{1}} c + ab) & \sigma_{23} b^2 + \sigma_{13} c^2 - \sigma_{12}^2 - a^2
\end{pmatrix},
\]

where

\[
D = \sigma_{23} b^2 + \sigma_{13} c^2 + \sigma_{12}^2 + a^2.
\]

and

\[
\sigma_{ij} = e^{x_{i+j}} 1 \leq i, j \leq 3.
\]

Therefore, it follows from Theorem 1 that

\[
\tilde{v} = (1, 0, 0)Y
\]

\[
\tilde{h} = -zY^0
\]

is a new solution for the IGWE. If we consider \( \tilde{v} \) given by (6) then we obtain the functions

\[
\varphi = \arctan \frac{\sqrt{(e^{2x(x_2 + x_3)} b^2 - e^{2x}(x_1 + x_2) c^2 - e^{2x}(x_1 + x_2) + a^2)^2 + 4 e^{2x}(x_1 + x_2)(e^{2x_{23}} bc + a)^2}}}{2 e^{x}(x_1 + x_2)(e^{2x_{23}} b - ac)}
\]

\[
\theta = \arctan \frac{e^{2x(x_2 + x_3)} b^2 - e^{2x}(x_1 + x_2) c^2 - e^{2x}(x_1 + x_2) + a^2}{2 e^{x}(x_1 + x_2)(e^{2x_{23}} bc + a)}
\]

where \( a, b, c \in \mathbb{R} \) and \( z \in \mathbb{R} \setminus \{0\} \), and \( \varphi, \theta \) satisfy the system of equations (7)-(12) with \( k = 0 \).

Similarly, starting with the trivial solution for the IGSGE, we solve (19) with \( B = C = 0 \), i.e.

\[
dY = \frac{z}{2} (E - YEY) + \frac{1}{2z} (QE - YEQY)
\]

(23)

for a given prescribed orthogonal matrix \( Y \) at the origin. Considering \( Y(0) \) as in (22), it is not difficult to see that the unique solution of (23) with this initial condition is given by
\[ Y = \frac{1}{\tilde{D}} \begin{pmatrix} \tilde{\delta}_{13}^2 b^2 - \tilde{\delta}_{13}^2 c^2 - \tilde{\delta}_{12}^2 + a^2 \\ 2\tilde{\delta}_{12}(e^{\tilde{c}z_k} bc + a) \\ -2\tilde{\delta}_{13}(e^{\tilde{c}z_k} b + ac) \end{pmatrix} \begin{pmatrix} 2\tilde{\delta}_{12}(e^{\tilde{c}z_k} bc + a) \\ -2\tilde{\delta}_{23}(e^{\tilde{c}z_k} c + ab) \\ 2\tilde{\delta}_{23}(e^{\tilde{c}z_k} b + ac) \end{pmatrix}, \]

where

\[ \tilde{D} = \tilde{\delta}_{23}^2 b^2 + \tilde{\delta}_{12}^2 c^2 + \tilde{\delta}_{12}^2 - a^2, \]

\[ \tilde{\delta}_{12} = e^{\lambda z_k + \delta z_k}, \]
\[ \tilde{\delta}_{13} = e^{\lambda z_k + \delta z_k}, \]
\[ \tilde{\delta}_{23} = e^{\delta (z_k + z_k)}, \]

and

\[ \lambda = \frac{1}{2}(z + \frac{1}{z}) \quad \delta = \frac{1}{2}(z - \frac{1}{z}). \]  

(24)

Therefore, it follows from Theorem 2 that

\[ \tilde{v} = (1, 0, 0)Y \]
\[ \tilde{h} = - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \delta & 0 \\ 0 & 0 & \delta \end{pmatrix} Y^0 \]

is a new solution for the IGSGE. If we consider \( \tilde{v} \) given by

\[ \tilde{v} = (\sin \varphi \sin \tilde{\theta}, \sin \varphi \cos \tilde{\theta}, \cos \varphi), \]

then we obtain the functions

\[ \tilde{\varphi} = \arctan \frac{\sqrt{e^{2\tilde{c}(x_k + z_k)} b^2 - e^{2(\lambda z_k + \delta z_k)} c^2 - e^{2(\lambda z_k + \delta z_k)} + a^2}^2 + 4e^{2(\lambda z_k + \delta z_k)}(e^{2\tilde{c}z_k} bc + a)^2}{2e^{\lambda z_k + \delta z_k}(e^{2\tilde{c}z_k} b - ac)} \]

\[ \tilde{\theta} = \arctan \frac{e^{2\tilde{c}(x_k + z_k)} b^2 - e^{2(\lambda z_k + \delta z_k)} c^2 - e^{2(\lambda z_k + \delta z_k)} + a^2}{2e^{\lambda z_k + \delta z_k}(e^{2\tilde{c}z_k} bc + a)} \]

where \( a, b, c \in \mathbb{R}, \quad z \in \mathbb{R} \setminus \{0\} \) and \( \lambda, \delta \) are given by (24). Then \( \varphi, \tilde{\theta} \) satisfy the system of equations (7)-(12) with \( k = -1 \).

In what follows, we fix the constants \( a, b, c \) and \( z \) and provide a graph of the solutions \( \varphi, \theta, \varphi, \tilde{\theta} \). We observe that the sign of the product \( abc \) affects the continuity of the functions.
3. Graphs associated to solutions of the IGWE and IGSGE.

Below we present the graph of solutions $\varphi(x_1, x_2, x_3)$, $\theta(x_1, x_2, x_3)$ for the IGWE and $\hat{\varphi}(x_1, x_2, x_3)$, $\hat{\theta}(x_1, x_2, x_3)$ of the IGSGE for fixed values of constants $a$, $b$, $c$ and $z$. Each column is a sequence of graphs in $\mathbb{R}^3$ where the observer is at a fixed position and one of the variables is constant assuming the sequence of values $-6, -3, 0, 3, 6$. The other variables vary in $[-8, 8]$. In the first column $x_1$ is fixed, in the second and third columns $x_2$ and $x_3$ are fixed respectively.

a) For $a = 1$, $b = -1$, $c = 1$ and $z = 4$, $\theta$ is not continuous and $\varphi$ has the following graph:
b) For \( a = b = c = 1 \) and \( z = 4 \), \( \varphi \) is not continuous and \( \theta \) has the following graph:
c) For $a = 1$, $b = -1$, $c = 1$ and $z = 0.2$, $\theta$ is not continuous and $\tilde{\theta}$ has the following graph:
d) For $a = b = c = 1$ and $z = 0.2$, $\tilde{\phi}$ is not continuous and $\tilde{\theta}$ has the following graph:
e) For $a = 0$, $b = -1$, $c = 1$ and $z = 4$, then $b$, $φ$ have the following graphs:
References


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