

SYMBOLIC COMPUTATIONS IN GEOMETRY

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Preface

The papers collected in this volume were presented at a concentrated week on *Symbolic Computations in Projective Geometry*, held at the Institute for Mathematics and its Applications, Minneapolis, October 12–16, 1987. This week was intended to study the use of coordinate-free algebraic methods in computational geometry. These methods include the Cayley or Grassmann algebra, the bracket algebra of invariants of the general linear group, and the Cayley-Menger algebra which is generated by Euclidean invariants. These structures, because of their coordinate-free (invariant) nature, lie closer to synthetic geometry than the usual algebra of coordinates versus a fixed frame of reference.

Papers I, V, and VI discuss Cayley factorization, which is the crucial step in interpreting algebraic expressions in projective geometry. Paper II examines some algebraic issues regarding the computer-aided coordinatization of combinatorial configurations, and in paper VII these methods are applied to a large class of previously unrealized configurations. Paper III deals with the logical underpinnings of invariant methods, and paper IV applies the algebra of Cayley-Menger determinants to computational problems in Euclidean geometry.

The volume is partially expository, but also contains many interesting new results, which will hopefully stimulate further research in this area. We wish to thank the I.M.A. for its support during our week.

Minneapolis, January 1988

Bernd Sturmfels and Neil White

Table of Contents

I. MULTILINEAR CAYLEY FACTORIZATION	<i>by Neil White</i>	1
I.1. Cayley algebra		1
I.2. Cayley factorization		4
I.3. Bracket algebra and straightening		7
I.4. Some lemmas		9
I.5. The Cayley factorization algorithm		11
I.6. Implementation		14
I.7. Questions		15
References		15
II. COMPUTATIONAL ALGEBRAIC GEOMETRY OF PROJECTIVE CONFIGURATIONS	<i>by Bernd Sturmfels</i>	17
II.1. Introduction		17
II.2. A construction method and more examples of configurations		21
II.3. On the existence of coordinatization algorithms		25
II.4. Complex versus real versus rational realizability		27
II.5. Variants of the Nullstellensatz		29
II.6. Computations of final polynomials and final syzygies		31
References		36
III. LOGIC AND INVARIANT COMPUTATION FOR ANALYTIC GEOMETRY		
<i>by Walter Whiteley</i>		38
III.1. The First Fundamental Theorem		38
III.2. The Second Fundamental Theorem		41
III.3. Proofs over algebraically closed fields		43
III.3.a. Open or universal theorems		43
III.3.b. Quantifiers over algebraically closed fields		47
III.4. Extensions to real closed fields		51
References		53

IV.	THE USE OF DISTANCES AS COORDINATES IN COMPUTER-AIDED PROOFS OF THEOREMS IN EUCLIDEAN GEOMETRY	<i>by Timothy Havel</i>	54
IV.1.	Introduction		54
IV.2.	Examples:		57
IV.2.1.	Isosceles bisectors		57
IV.2.2.	The law of parallelograms		58
IV.2.3.	Simpson's theorem		60
IV.2.4.	Topology of the equilateral pentagon linkage		63
	References		67
V.	TOWARDS NONLINEAR CAYLEY FACTORIZATION	<i>by Henry Crapo</i>	69
V.1.	Introduction		69
V.2.	Example: the projective condition for a polyhedral scene		70
V.3.	Calotte conditions		77
V.4.	A simpler example		80
VI.	ON THE SYNTHETIC FACTORIZATION OF HOMOGENEOUS INVARIANTS		
	<i>by Bernd Sturmfels and Walter Whiteley</i>		82
VI.1.	Introduction and statement of the result		82
VI.2.	Proof of the factorization theorem		85
	References		91
VII.	RATIONAL REALIZATIONS OF 11_3 - and 12_3 -CONFIGURATIONS		
	<i>by Bernd Sturmfels and Neil White</i>		92
VII.1.	The main result		92
VII.2.	An example: the "cyclic" configuration $(12_3)_{228}$		93
	References		96
VII.Appendix A.	Rational coordinatizations for the configurations 11_3		97
VII.Appendix B.	Rational coordinatizations for the configurations 12_3		101