A DETERMINISTIC APPROACH TO OPTIMAL STOPPING
WITH APPLICATION TO A PROPHET INEQUALITY

By

Mark H.A. Davis

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A DETERMINISTIC APPROACH TO OPTIMAL STOPPING
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MARK H.A. DAVIS*

Abstract. This paper concerns the optimal stopping problem of maximizing \( EY_\tau \) over the set \( \mathcal{M}_0 \) of all finite-valued stopping times \( \tau \) of a filtration \( (\mathcal{F}_k) \), where \( Y_k \) is a non-negative adapted process. It is shown that \( \sup_{\tau \in \mathcal{M}_0} EY_\tau = E[\sup_{k \in Z_+} (Y_k + \lambda_k)] \) for some process \( (\lambda_k) \) satisfying \( E\lambda_\tau = 0 \) for all \( \tau \in \mathcal{M}_0 \). Thus \( (\lambda_k) \) is the Lagrange multiplier corresponding to the “non-anticipativity constraint” that \( \tau \) is a stopping time (rather than a general non-adapted random time). The basic “prophet inequality” of Krenkel and Sucheston is shown to follow easily from this result.

Key words. Optimal stopping, Snell envelope, discrete-parameter martingales, Lagrange multipliers, randomized stopping rules

AMS(MOS) subject classifications. 62L15, 60G40, 60G42

1. Introduction. Let \((\Omega, \mathcal{F}, P)\) be a complete probability space and \((\mathcal{F}_k)_{k \in Z_+}\) be a filtration, i.e. \( \mathcal{F}_k \) is a sub \( \sigma \)-field of \( \mathcal{F} \) for each \( k \in Z_+ = \{0, 1, 2, \ldots \} \), \( \mathcal{F}_k \subset \mathcal{F}_{k+1} \) and each \( \mathcal{F}_k \) contains all the null sets of \( \mathcal{F} \). \( \mathcal{M}_n, \mathcal{M}_n^N \) will denote the sets of \( \mathcal{F}_k \)-stopping times \( \tau \) such that, respectively, \( P[n \leq \tau < \infty] = 1 \) and \( P[n \leq \tau \leq N] = 1 \). Now let \((Y_k)_{k \in Z_+}\) be a stochastic process such that \( Y_k \) is \( \mathcal{F}_k \)-measurable for each \( k \in Z_+ \), \( Y_k \geq 0 \) a.s. and

\[
M := E \left[ \sup_{k \in Z_+} Y_k \right] < \infty.
\]

The general problem of optimal stopping, as discussed in, for example, \([4]-[5]\), is to compute

\[
V = \sup_{\tau \in \mathcal{M}_0} EY_\tau
\]

and to find – if one exists – a stopping time \( \hat{\tau} \) such that \( V = EY_{\hat{\tau}} \).

The problem is of course trivial if \( \mathcal{F}_0 = \mathcal{F} \). Then any \( Z_+ \)-valued random variable is in \( \mathcal{M}_0 \) and \( V = M \), defined by (1). Optimal times take the form \( \hat{\tau}(w) \in \arg\max_k Y_k(w) \) (assuming the maximum is attained) and, as pointed out by Neveu \([4]\) the smallest such time can be expressed as \( \hat{\tau}(w) = \min\{k : Y_k(w) = Z_k(w)\} \) where \( Z_k(w) = \sup_{j \geq k} Y_j(w) \). The essence of the problem is therefore that in general \( \mathcal{F}_k \neq \mathcal{F} \) and the times \( \tau \) must satisfy the non-activativity requirement \( \{\tau \leq k\} \in \mathcal{F}_k, k \in Z_+ \) which can be thought of as a linear equality constraint, i.e. \( P[\tau \leq k | \mathcal{F}_k] = I_{[\tau \leq k]} \). It can be enforced by introducing a suitable Lagrange multiplier process \( \lambda_k \). This idea, originally due to Wets \([6]\) in the context of stochastic programming, has been pursued in a control theory setting in, for example,
In this paper we will show that there is a unique choice of multipliers $\lambda_k$ such that $V$ defined by (2) is given by

$$V = E \left[ \sup_{k \in \mathbb{N}} (Y_k + \lambda_k) \right],$$

while for any stopping time $\tau$, $E[\lambda_\tau] = 0$, so that $E[Y_\tau + \lambda_\tau] = EY_\tau$. Thus optimal or $\epsilon$-optimal stopping times are actually pathwise maximizers of $Y_k + \lambda_k$. $\lambda_k$ is given explicitly by $\lambda_k = M_\infty - M_k$, where $M_k$ is the martingale component in the Doob decomposition of the Snell envelope $Z_k$ of $Y_k$. The Snell envelope is described in Section 2 below. Section 3 discusses the finite-horizon case, where it becomes evident from a dynamic programming argument why the multiplier takes the form it does. The general case is treated in Section 4, the main result being Theorem 1 in that section. This is applied in Section 5 to derive the basic "prophet inequality" $M \leq 2V$ of Krengel and Sucheston, valid when the $Y_k$'s are independent random variables.

2. The Snell envelope. This section summarizes information that can be found in chapter VI of Neveu [4]. The Snell envelope of $Y_k$ is the process $Z_k$ defined by

$$Z_k = \text{ess sup}_{\tau \in \mathcal{M}_k} E[Y_\tau | \mathcal{F}_k].$$

$Z_k$ is integrable in view of condition (1). It is a supermartingale, is the smallest non-negative supermartingale dominating $Y_k$, and satisfies the recursion $Z_k = Y_k \lor E[Z_{k+1} | \mathcal{F}_k]$. The main general result of optimal stopping theory is that an optimal stopping time exists if and only if the stopping time $\sigma_0 := \min \{ k : Y_k = Z_k \}$ is a.s. finite (i.e. belongs to $\mathcal{M}_0$), and then $\sigma_0$ is the smallest optimal time. Even if no optimal time exists, $\sigma_\epsilon := \min \{ k : Z_k - Y_k \leq \epsilon \}$ is in $\mathcal{M}_0$ for any $\epsilon > 0$ and satisfies $EY_{\sigma_\epsilon} \geq V - \epsilon$, where $V$ is given by (2).

For finite horizon problems the Snell envelope can be defined in an algorithmic manner as follows: let

$$Z_N^N = Y_N$$

$$Z_k^N = Y_k \lor E[Z_{k+1}^N | \mathcal{F}_k], \quad k = N - 1, \ldots, 0.$$}

Then

$$Z_k^N = \text{ess sup}_{\tau \in \mathcal{M}_k^N} EY_\tau.$$}

An optimal stopping time $\hat{\sigma}^N \in \mathcal{M}_0^N$ always exists and is given by

$$\hat{\sigma}^N = \min \{ k \leq N : Y_k = Z_k^N \}.$$
Being a non-negative supermartingale, $Z_k$ has the Doob decomposition $Z_k = M_k - A_k$ where $M_k$ is a martingale and $A_k$ is a predictable increasing process (i.e. $A_k$ is $\mathcal{F}_{k-1}$ measurable, $k = 1, 2, \ldots$). $M_k$ and $A_k$ are given recursively by

\begin{align*}
M_k &= M_{k-1} + (Z_k - E[Z_k|\mathcal{F}_{k-1}]), \quad M_0 = Z_0 \\
A_k &= A_{k-1} + (Z_{k-1} - E[Z_{k-1}|\mathcal{F}_{k-1}]), \quad A_0 = 0
\end{align*}

3. **The finite horizon case.** Here we consider maximizing $EY_\tau$ over $\tau \in \mathcal{M}_0^N$. It is instructive to consider an enlarged class of randomized strategies $\mathcal{R}^N$ as discussed by, for example, Shiryaev [5]. An element of $\mathcal{R}^N$ is an adapted sequence $\theta = \{\theta_0, \ldots, \theta_N\}$ with $\theta_k \in [0, 1], k = 0, \ldots, N-1$ and $\theta_N \equiv 1$. One flips a coin with heads probability $\theta_k$ in order to decide whether to stop the process at time $k$ given that it was not stopped earlier. An ordinary stopping time $\tau$ is the special case $\theta_k = I_{\{\tau = k\}}$. The stopping time distribution is then $\rho_k, k = 0, \ldots, N$ given by

$$\rho_k = \theta_k \prod_{j=0}^{k-1} (1 - \theta_j)$$

and the corresponding reward is

$$J(\theta) = E \sum_{k=0}^{N} Y_k \rho_k,$$

so that $J(\theta) = EY_\tau$ when $\theta_k = I_{\{\tau = k\}}$. It is easily shown that the available reward is not increased by enlarging the class of strategies in this way, i.e.

$$\sup_{\theta \in \mathcal{R}^N} J(\theta) = \sup_{\tau \in \mathcal{M}_0^N} EY_\tau.$$ 

It is convenient to define

$$\zeta_k = \prod_{j=0}^{k-1} (1 - \theta_j) \quad k = 1, \ldots, N$$

and $\zeta_0 = 1$, so that

\begin{align*}
(7) \\
\zeta_{k+1} &= (1 - \theta_k)\zeta_k, \quad \zeta_0 = 1 \\
\rho_k &= \zeta_k \theta_k
\end{align*}

and the reward is

$$J(\theta) = E \sum_{k=0}^{N} Y_k \zeta_k \theta_k.$$
We now wish to consider *pathwise* optimization, relaxing the adaptedness constraint but introducing a multiplier process. Thus fix \( w \in \Omega \), write \( Y_k := Y_k(w) \) and consider maximizing \( K(w, \theta) \) given by

\[
K(w, \theta) = \sum_{k=0}^{N} (Y_k + \lambda_k) \zeta_k \theta_k
\]

over arbitrary sequences \( \theta \), where \( \zeta_k \) is given by (7). This is an optimal control problem, and elementary application of dynamic programming given the following result.

**Lemma 1.** Let \( V_k(\zeta) = \max_{\theta_k, \ldots, \theta_{N-1}} \sum_{j=k}^{N} (Y_j + \lambda_j) \zeta_j \theta_j \) subject to (7) with \( \zeta_k = \zeta \). Then \( V_k(\zeta) = \beta_k(\lambda) \zeta \) where

\[
\begin{align*}
\beta_N(\lambda) &= Y_N + \lambda_N \\
\beta_k(\lambda) &= (Y_k + \lambda_k) \lor \beta_{k+1}(\lambda) \quad k = N - 1, \ldots, 0.
\end{align*}
\]

(8)

This shows that, as before, “randomization” has no effect and \( \beta_0(\lambda) = \max_{0 \leq k \leq N} (Y_k + \lambda_k) \).

Most of the remaining argument is based on the simple identity \( a \lor b = b + [a - b]^+ \), where \( [c]^+ = c \lor 0 \). We write \( E^k X := E[X | \mathcal{F}_k] \) for integrable r.v.’s \( X \). Take for example \( k = N - 1 \) in (8) and assume \( \lambda_N = 0 \), to give

\[
\beta_{N-1}(\lambda) = Y_N + [Y_{N-1} + \lambda_{N-1} - Y_N]^+
\]

If we choose \( \lambda_{N-1} = Y_N - E^{N-1}Y_N \) then the optimal decision at time \( N - 1 \) is \( \hat{\theta}_{N-1} = I_{\{Y_{N-1} \geq E^{N-1}Y_N\}} \) which coincides with the best non-anticipative decision. Further,

\[
E \sum_{j=N-1}^{N} (Y_j + \lambda_j) \zeta_j \theta_j = E \sum_{j=N-1}^{N} Y_j \zeta_j \theta_j
\]

for any adapted \( \theta \), and these choices of \( \lambda_{N-1}, \lambda_N \) are the only ones for which these two properties hold. The general result is as follows.

**Proposition 1.** Let \( Z^N_k \) be the Snell envelope for the \( N \)-horizon problem, defined by (5). Define

\[
\begin{align*}
\alpha_N &= Y_N, \quad \lambda_N = 0 \\
\alpha_k &= \alpha_{k+1} + [Y_k - E^k Z^N_{k+1}]^+, \quad k = N - 1, \ldots, 0 \\
\lambda_k &= \alpha_{k+1} - E^k \alpha_{k+1}, \quad k = N - 1, \ldots, 0
\end{align*}
\]

(9) (10)

Then, in the notation of Lemma 1, \( \alpha_k = \beta_k(\lambda) \),

\[
E[\alpha_k | \mathcal{F}_k] = Z^N_k,
\]

(11)
and the strategy \( \hat{\theta}_k = \mathbb{I}_{\{\pi_N(w) = k\}} \) maximizes \( K(w, \theta) \) for almost every \( w \).

**Proof.** This is proved by a simple induction argument. For example, to verify (11), suppose that \( E[\alpha_{k+1}|\mathcal{F}_{k+1}] = Z^N_{k+1} \); then from (9)

\[
E^k \alpha_k = E^k \alpha_{k+1} + [Y_k - E^k Z^N_{k+1}]^+
\]

\[
= E^k Z^N_{k+1} + [Y_k - E^k Z^N_{k+1}]^+
\]

\[
= (E^k Z^N_{k+1}) \lor Y_k = Z^N_k,
\]

showing that (11) holds for all \( k \), since clearly it holds when \( k = N \). \( \square \)

It is possible to express (9) in more convenient form. Denote temporarily \( Y := Y_k, Z := Z^N_k \) and \( E := E^k Z^N_{k+1} \). Then \( Z = Y \lor E = Y + [E - Y]^+ \) and hence \([Y - E]^+ = [E - Y]^- = [E - Y]^+ - (E - Y) = (Z - Y) - (E - Y) = Z - E\), so that we have

\[
\alpha_N = Z^N_N, \quad \alpha_k = \alpha_{k+1} + Z^N_k - E[Z^N_{k+1}|\mathcal{F}_k]
\]

or

\[
\alpha_k = \sum_{j=k}^{N-1} (Z^N_j - E^j Z^N_{j+1}) + Z^N_N.
\]

**4. The general case.** We now resume consideration of the infinite-horizon problem. Recall that \( Z_k \) denotes the Snell envelope, defined by (4).

**Lemma 2.** \( E\{\sum_{k=0}^{\infty} (Z_k - E^k Z_{k+1})\} < \infty \); in particular

\[
\sum_{n=0}^{\infty} (Z_k - E^k Z_{k+1}) < \infty \text{ a.s.}
\]

**Proof.** Each term in the sum is a.s. non-negative and the \( n \)'th partial sum satisfies

\[
E \sum_{k=0}^{N} (Z_k - E^k Z_{k+1}) = E(Z_0 - Z_{N+1}) \leq EZ_0 < \infty.
\]

The result follows by monotone convergence. \( \square \)

Since \( Z_k \) is a non-negative supermartingale, the limit \( Z_\infty = \lim_{k \to \infty} Z_k \) exists a.s. and in \( L_1 \), and by analogy with (9), (10) we now define

\[
\alpha_k = \sum_{j=k}^{\infty} (Z_j - E^j Z_{j+1}) + Z_\infty
\]

(12)

\[
\lambda_k = \alpha_k - Z_k(= \alpha_{k+1} - E^k Z_{k+1}).
\]

(13)

We see from (6) that \( \lambda_k \) can be expressed as \( \lambda_k = M_\infty - M_k \). This brings us to the main result.
Theorem 1. Suppose that \( Y_k \geq 0 \) and that condition (1) is satisfied. Define \( \alpha_k, \lambda_n \) by (12), (13). so that \( \lambda_k = M_\infty - M_k \) where \( M_k \) is the martingale appearing in the Doob decomposition of the Snell envelope \( Z_k \) of \( Y_k \). Then

\[
\sup_{\tau \in M_0} EY_\tau = E \left\{ \sup_{k \in Z_+} (Y_k + \lambda_k) \right\},
\]

while \( E\lambda_\tau = 0 \) for any stopping time \( \tau \in M_0 \).

Proof. It follows as in the previous section that \( \alpha_k \) defined by (12) satisfies

\[
\alpha_k = \alpha_{k+1} + [Y_k - E^k Z_{k+1}]^+
\]

and hence by a dynamic programming argument that

\[
\alpha_k = \sup_{j \geq k} (Y_j + \lambda_j)
\]

where \( \lambda_j \) is defined by (13). By monotone convergence, and the a.s. and \( L_1 \) convergence of \( Z_k \) to \( Z_\infty \),

\[
E[\alpha_k | \mathcal{F}_k] = \lim_{N \rightarrow \infty} \left[ \sum_{j=k}^{N} (Z_j - E^j Z_{j+1}) + Z_\infty | \mathcal{F}_k \right]
\]

\[
= Z_k + \lim_{N \rightarrow \infty} E[Z_\infty - Z_{N+1} | \mathcal{F}_k]
\]

\[
= Z_k.
\]

In particular (a) \( E\alpha_0 = EZ_0 \), from which (14) follows, and (b) \( E[\lambda_k | \mathcal{F}_k] = 0 \), so that for any \( \tau \in M_0 \)

\[
E\lambda_\tau = E \left\{ \sum_{k=0}^{\infty} I_{\{\tau = k\}} \lambda_k \right\}
\]

\[
= E \left\{ \sum_{k=0}^{\infty} I_{\{\tau = k\}} E[\lambda_k | \mathcal{F}_k] \right\} = 0.
\]

This completes the proof. \[\square\]

5. Prophet Inequalities. “Prophet inequalities” are statements of the form \( M \leq cV \), where \( M, V \) are defined by (1), (2), \( c \) is a constant and the inequality holds for all probability measures \( P \) in some set \( \mathcal{C} \). The interpretation then is a “prophet” possessed of complete clairvoyance can obtain a reward which is only \( c \) times that of a player restricted to non-anticipative strategies. An excellent survey of results and techniques in this area can be found in Hill and Kertz [2]. The following classical result of Krengel, Sucheston and Garling [3] follows easily from Theorem 1.
Theorem 2. Suppose that \( Y_k \geq 0 \) for all \( k \), that condition (1) holds and that for each \( k \in \mathbb{Z}_+ \) and \( j > k \) the random variable \( Y_j \) is independent of \( \mathcal{F}_k \). Then \( M \leq 2V \).

Remark. The conditions imply that the r.v.'s \( Y_k \) are mutually independent and constitute a semiamart in the terminology of [4]. It is clear that under these conditions

\[
V = \sup_{\tau \in \mathcal{M}_0'} EY_{\tau}
\]

where \( \mathcal{M}_0' \) denoted the set of finite-valued stopping times of the natural filtration

\[
\mathcal{F}_k' = \sigma\{Y_0, Y_1, \ldots, Y_k\}.
\]

Proof. Under the independence condition stated, it follows from the property \( Z_k = Y_k \lor E(Z_{k+1}|\mathcal{F}_k) \) that there is a sequence of constants \( v_0 \geq v_1 \geq v_2 \cdots \geq 0 \) such that

\[
Z_k = Y_k \lor v_{k+1}
\]

and

\[
v_k = E[Y_k \lor v_{k+1}]
\]

Thus, from (12) and (13)

\[
Y_k + \lambda_k = \sum_{j=k}^{\infty} [Y_j - v_{j+1}]^+ + Y_k - Y_k \lor v_{k+1} + Z_\infty
\]

(15)

\[
= \sum_{j=k+1}^{\infty} [Y_j - v_{j+1}]^+ + Y_k - v_{k+1} + Z_\infty,
\]

where the second equality follows from the representation \( Y_k \lor v_{k+1} = v_{k+1} + [Y_k - v_{k+1}]^+ \). Thus from (14) and using (15)

\[
V = E\{ \sup_{k \in \mathbb{Z}_+} (Y_k + \lambda_k) \}
\]

\[
\geq E\{ \sup_{k \in \mathbb{Z}_+} (Y_k - v_{k+1}) \}
\]

\[
\geq E\{ \sup_{k \in \mathbb{Z}_+} (Y_k - v_0) \}
\]

\[
= M - V
\]

Thus \( M \leq 2V \), as claimed. \( \Box \)

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