SOURCE LOCALIZATION PROCESSING IN PERTURBED WAVEGUIDES

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Abstract

In this paper, we combine the matched-field processing with the boundary integral equation method from the scattering theory to study a sound source localization problem in a perturbed shallow ocean. We assume that there is an inclusion embedded in a shallow water waveguide. Continuous wave (CW), produced by a sound source, is scattered by the inclusion and then received by a hydrophone array. Because the symmetry of the waveguide has been destroyed by the existence of the inclusion, a proper procedure is required to avoid the mismatching. We present a numerical scheme which makes use of the separation of the source and the detection array, and greatly reduces the computation. A numerical simulation using this method is presented.

1 Introduction

Localization of the acoustic source in waveguides has been studied by many authors in recent years [1] [2] [7][8][9]. One of the most significant progress is probably the “matched-field” method which is proposed by Bucker [2] in 1976. The main idea of the matched-field processing method is, outlined by the title of Bucker’s paper, “use of calculated wave field and matched field detection to locate sound source”. The matched-field processing is usually performed in either “phone space” (matching the total field received by each hydrophone) or in “mode space” (matching the resolved modes).

On the other hand, the classical inverse scattering theories which usually involve more mathematics have been rapidly developed in about the same time [3] [10] [4]. The basic idea of the inverse scattering theory is based on the physical idea of scattering one or more “plane waves” off the unidentified inclusion and then trying to identify the shape of the inclusion or other properties from its far-field patterns. Recently, Gilbert and Xu have generalized this idea to the direct and inverse scattering problems in a shallow ocean (ref. [5][6][11][12]).

However, there is a concern remained, in particular from the engineering’s aspect, in the inverse scattering theory in shallow ocean. That is, a “complete set of data” is required in order to find a reasonable solution. Unfortunately, this “complete data” is not always available in practice. It raises a question, that is, if we can find a compliment between the inverse scattering theory and the matched-field signal processing so that we can use less detected information to estimate the unknown object, or localize the sound source in a more complicate environment.

In this paper, we combine the matched-field processing with the boundary integral equation method from the scattering theory to study a sound source localization problem in a perturbed shallow ocean. We assume that there is an inclusion embedded in a shallow water waveguide. Continuous wave (CW), produced by a sound source, is scattered by the inclusion and then received by a hydrophone array (Figure 1). Because the symmetry of the waveguide has been destroyed by the existence of the inclusion, the methods presented in [1] [2] [7][8][9] are no longer suitable.

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section 2, we will formulate the problem and present the theoretical consideration. Then in section 3, some numerical examples are presented. A numerical method for the boundary integral equation which is essential in our computation is included in the Appendix.

2 Modeling and methodology

2.1 Modeling

The synthetic modeling of the perturbed waveguide is depicted in figure 1.

We denote the waveguide with depth $d$ as $\mathbb{R}^2_d = \{(x_1, x_2) | -\infty < x_1 < \infty, 0 \leq x_2 \leq d\}$. An inclusion which is a bounded region located in the waveguide is denoted as $\Omega$. For the sake of exposure of our method, we shall assume that the inclusion has a sound-soft boundary $\partial\Omega$. A time-harmonic acoustic source locates at $x^s = (x^s_1, x^s_2)$. The hydrophone array consists of $L$ hydrophones at $x^l = (x^l_1, x^l_2), l = 1, 2, \cdots, L$. The time-harmonic wave, radiated from $x^s$ and scattered by $\Omega$, propagates outward to $|x_1| \to \infty$. Let $p(x; x^s)$ be the acoustic pressure at $x = (x_1, x_2)$, emitted from the acoustic source at $x^s$, and $k = 2\pi f/c$ be the wave number, where $f$ is the frequency and $c$ is the speed of the time-harmonic acoustic wave. If the water waveguide has a pressure released surface $x_2 = 0$ and a rigid bottom $x_2 = d$, then the propagation of the outgoing wave is governed by the following system:

$$
\begin{align*}
\Delta p(x; x^s) + k^2 p(x; x^s) &= \delta(x_1 - x^s_1)\delta(x_2 - x^s_2), \ x = (x_1, x_2) \in \mathbb{R}^2_d \setminus \bar{\Omega}, \\
p(x_1, 0; x^s) &= 0, \quad \frac{\partial p}{\partial x_2}(x_1, d; x^s) = 0, \\
p(x; x^s) &= 0 \text{ for } x \in \partial\Omega.
\end{align*}
$$

Moreover, $p(x; x^s)$ satisfies an outgoing radiating condition, i.e., for $|x_1| \to \infty$, $p(x; x^s)$ has an expansion

$$
p(x_1, x_2) = \sum_{n=1}^{\infty} p_n \sin\left[(n - \frac{1}{2})\frac{\pi x_2}{d}\right] e^{ik_n|x_1|},
$$

where $k_n = [k^2 - (n - \frac{1}{2})^2 \frac{x^2}{d^2}]^{1/2}$ is the horizontal wavenumber, and the coefficients $p_n$ depend on $x^s$ and the sign of $x_1$.

Now we can state our source localization problem as following: given the acoustic pressure at points $x^l, l = 1, 2, \cdots, L$ in the aforementioned perturbed waveguide, to estimate the location of the sound source $x^s$.

2.2 Construction of the propagator

The propagating acoustic wave emitting from a point source at $x^s$ (which is called propagator) can be constructed in the following way.

Let $p_0(x; x^s)$ be the propagator in an unperturbed waveguide, i.e., $p_0(x, x^s)$ satisfies

$$
\begin{align*}
\Delta p_0(x; x^s) + k^2 p_0(x; x^s) &= \delta(x_1 - x^s_1)\delta(x_2 - x^s_2), \ x = (x_1, x_2) \in \mathbb{R}^2_d \\
p_0(x_1, 0; x^s) &= 0, \quad \frac{\partial p_0}{\partial x_2}(x_1, d; x^s) = 0,
\end{align*}
$$

and $p_0(x; x^s)$ is outgoing. By separation of variables, we can represent $p_0(x; x^s)$ as

$$
p_0(x; x^s) = \sum_{n=1}^{\infty} \frac{i}{\pi k a_n} \sin\left[(n - \frac{1}{2})\frac{\pi x_2}{d}\right] \sin\left[(n - \frac{1}{2})\frac{\pi x^s_2}{d}\right] e^{ik_n|x_1 - x^s_1|},
$$

(2.7)
where
\[ a_n = \left[ 1 - \frac{(2n - 1)2^{1/2}}{4k^2d^2} \right]^2. \]  
(2.8)

We write the propagator in the perturbed waveguide as
\[ p(x; x^*) = p_0(x; x^*) + p_1(x; x^*). \]  
(2.9)

Then \( p_1 = p - p_0 \) is a solution of the problem
\[ \Delta p_1(x; x^*) + k^2 p_1(x; x^*) = 0, \quad x \in \mathbb{R}_d^2 \setminus \Omega, \]  
(2.10)
\[ p_1(x_1, 0; x^*) = 0, \quad \frac{\partial p_1}{\partial x_2}(x_1, d; x^*) = 0, \]  
(2.11)
\[ p_1(x; x^*) = -p_0(x; x^*) \text{ for } x \in \partial \Omega, \]  
(2.12)
and \( p_1(x; x^*) \) is out-going wave. The physical meaning of this problem is that an incident wave \( p_0 \) incidents upon the inclusion \( \Omega \) and produces the scattered wave \( p_1 \). The propagator \( p \) is the composition of the incident wave \( p_0 \) and the scattered wave \( p_1 \).

The scattered wave \( p_1 \) can be constructed by boundary integral equation method. We defining a double layer potential
\[ p_1(x; x^*) = \int_{\partial \Omega} \frac{\partial p_0(x; y)}{\partial \nu_y} \psi(y; x^*) d\sigma_y, \quad \text{for } x \in \mathbb{R}_d^2 \setminus \overline{\Omega}, \]  
(2.13)
where \( \psi \) is the solution of the boundary integral equation
\[ \psi(x; x^*) + 2 \int_{\partial \Omega} \frac{\partial p_0(x; y)}{\partial \nu_y} \psi(y; x^*) d\sigma_y = -2p_0(x, x^*), \quad \text{for } x \in \partial \Omega. \]  
(2.14)

If \( k \) is not an eigenvalue of the interior Neumann problem in \( \Omega \), then (2.14) has a unique solution. Symbolically we denote the boundary integral equation (2.14) as
\[ \psi + K \psi = -2p_0, \]  
(2.15)
where \( K \) is the integral operator
\[ K \psi(x; x^*) := 2 \int_{\partial \Omega} \frac{\partial p_0(x; y)}{\partial \nu_y} \psi(y; x^*) d\sigma_y, \quad \text{for } x \in \partial \Omega. \]  
(2.16)

If \( k \) is not an eigenvalue of the interior Neumann problem in \( \Omega \), then \( I + K \) is invertible. We can write
\[ \psi(x; x^*) = -2(I + K)^{-1}p_0(x; x^*), \]  
(2.17)
and
\[ p(x; x^*) = p_0(x; x^*) - \int_{\partial \Omega} \frac{\partial p_0(x; y)}{\partial \nu_y} (I + K)^{-1}p_0(y; x^*) d\sigma_y, \quad \text{for } x \in \mathbb{R}_d^2 \setminus \overline{\Omega}. \]  
(2.18)

By the assumption of the boundedness of the inclusion \( \Omega \), we know that for \( |x_1| \) large enough (say, \( |x_1| > x_0 \) for some constant \( x_0 \)), \( p(x; x^*) \) is expressed by a summation of normal modes:
\[ p(x; x^*) = \sum_{n=1}^{\infty} A_n(x^*) \sin(\frac{1}{2} \frac{\pi}{d} x_2) e^{i k_n |x_1|}, \]  
(2.19)
where \( A_n(x^*) \) is the modal amplitude given by
\[ A_n(x^*) = \frac{i}{\pi k a_n} \left( \sin(\frac{1}{2} \frac{\pi}{d} x_2) e^{-i k_n x_1 \text{sgn}(x_1)} \right. \]
\[ - \left. \int_{\partial \Omega} \frac{\partial (\sin(\frac{1}{2} \frac{\pi}{d} y_2) e^{-i k_n y_1 \text{sgn}(x_1)})}{\partial \nu_y} (I + K)^{-1}p_0(y; x^*) d\sigma_y \right), \quad \text{for } |x_1| > x_0. \]  
(2.20)

An approximate boundary integral equation method for the numerical solution of (2.15) is outlined in the Appendix. For more detail discussion of this method, readers could refer [13].
2.3 Construction of estimators

Using the representations for the propagator and its modal amplitude, we now construct the estimators in both phone space and mode space.

**Estimator in phone space** Let \( \{ p_{m1}^* \} \) be the detected data set consisting of the acoustic pressure field \( p_{m1}^* \) sampled on each hydrophone located at \((x_1^m, x_2^m)\), \(m = 1, 2, \cdots, M; l = 1, 2, \cdots, L\). The estimator in phone space is defined as follows:

\[
F_p(x_1^s, x_2^s) = \left( \sum_{l=1}^{L} \sum_{m=1}^{M} \left| p(x_1^m, x_2^m; x_1^s, x_2^s) - p_{m1}^* \right|^2 \right)^{-1}, \quad (2.21)
\]

where \( p(x_1^m, x_2^m; x_1^s, x_2^s) \) is the calculated acoustic pressure field at \((x_1^m, x_2^m)\). It can be computed by (2.18) for given \( x^s \) using the method outlined in the Appendix.

**Estimator in mode space** Let \( \{ p_l^* \} \) be the data set consisting of the acoustic pressure field \( p_l^* \) sampled on each hydrophone located at \( x_1^l, l = 1, 2, \cdots, L \) of a vertical array. Using a mode filtering approach, (for example, by least-squares best-fitting, damped least-squares best-fitting, or singular value decomposition), we obtain a set of complex modal amplitudes \( A_n^*, n = 1, \cdots, N \). The estimator in mode space is defined as follows:

\[
F_m(x_1^s, x_2^s) = \left( \sum_{n=1}^{N} \left| A_n(x_1^s, x_2^s) - A_n^* \right|^2 \right)^{-1}, \quad (2.22)
\]

where \( A_l(x_1^l, x_2^l) \) is the calculated complex modal amplitudes. It can be computed by (2.20) for each given \( x^s \).

2.4 Approximation of the estimators

The estimators presented in the last section provide a tool to localize acoustic source in a perturbed shallow ocean. To scan an area, we may compute the estimator for each chosen point \( x^s \) in the area. If the point \( x^s \) is close to the real location of the acoustic source, the estimator may appear as a large number. For a uniform searching in a rectangular area, this scheme requires a source searching number \( N_1 \times N_2 \), where \( N_1 \) and \( N_2 \) are the gridding numbers of range and depth respectively. We know that in a stratified ocean, some effective source localization processing methods have been discovered. For example, in [9], Shang presented a high-resolution method of source localization processing in mode space, which requires only \( N_1 + N_2 \) searching number. But in a perturbed waveguide, it is no longer proper to separate the depth search and the range search, because the separation of variables is no longer valid in the whole waveguide.

Fortunately, the representation (2.18) can be used to separate the source location and the detecting locations. This will greatly reduce the computation load.

In view of (2.7), we can rewrite (2.18) as (for \( x_1^s < y_1 < x_1 \))

\[
p(x; x^s) = \sum_{n=1}^{\infty} \frac{i}{\pi k a_n} \sin \left[ \left( n - \frac{1}{2} \right) \frac{\pi x_2^s}{d} \right] e^{-ik_n x_1^s} \left\{ \sin \left[ \left( n - \frac{1}{2} \right) \frac{\pi y_2}{d} \right] e^{ik_n y_1} \right. \\
- \left. \int_{\partial \Omega} \frac{\partial p_l(x, y)}{\partial n}(I + K)^{-1} \left( \sin \left[ \left( n - \frac{1}{2} \right) \frac{\pi y_2}{d} \right] e^{ik_n y_1} \right) d\sigma_y \right\}. \quad (2.23)
\]

For other cases of \( x_2^s, y_1, \) and \( x_1 \), we can get a similar representation with a proper change of the signs of \( x_2^s, y_1, \) and \( x_1 \).

Hence, we can approximate \( p(x; x^s) \) by

\[
p_N(x; x^s) = \sum_{n=1}^{N} \frac{i}{\pi k a_n} B_n(x_1, x_2) \sin \left[ \left( n - \frac{1}{2} \right) \frac{\pi x_2^s}{d} \right] e^{-ik_n x_1^s}, \quad (2.24)
\]
where
\[
B_n(x_1, x_2) = \sin \left( n - \frac{1}{2} \frac{\pi x_2}{d} \right) e^{ikan x_1} - \int_{\partial \Omega} \frac{\partial p_0(x, y)}{\partial \nu_y} (I + K)^{-1} \left( \sin \left( n - \frac{1}{2} \frac{\pi y_2}{d} \right) e^{ikan y_1} \right) d\sigma_y,
\]
and \(N\) is a properly chosen positive number. Note that \(B_n(x_1, x_2)\) does not depend on \(x^s\). Therefore, we can compute \(p_N(x_1, x_2)\) in two separated steps:

1. Compute \(B_n(x_1, x_2)\) for given \(l = 1, 2, \cdots, L\). First we solve the integral equation (2.15) where the right-hand-side is changed to \(-2 \sin \left( n - \frac{1}{2} \frac{\pi x_2}{d} \right) e^{ikan x_1}\), with \(n = 1, 2, \cdots, N\). Then substituting the solution \((I + K)^{-1}(-2 \sin \left( n - \frac{1}{2} \frac{\pi x_2}{d} \right) e^{ikan x_1})\) into (2.25), we obtain the \(B_n(x_1)\) for \(n = 1, 2, \cdots, N\). This calculation requires to solve the integral equation for only \(N\) times.

2. Compute \(p_N(x_1, x_2)\) for given \(x^s\). After \(B_n(x_1, x_2)\) are obtained, \(p_N(x_1, x_2)\) can be calculated economically using (2.24) for large number of source searching \(x^s\).

3 Computer simulation

Computer simulations using the aforementioned method are carried out on Cray2 of Minnesota Supercomputer Center. In this section we present two examples from our computations.

Example 1: Vertical hydrophone array

The synthetic modeling of the computer simulation is depicted in figure 2.

We assume the waveguide has depth of 100 meters. The sound speed of water is assumed to be 1500m/s. An acoustic source \(S\) located at \((-500/\pi, 100/\pi)\) emitting a time-harmonic wave at the frequency \(f = 30Hz\). The hydrophone array is arranged vertically at \((600/\pi, 2.5j), j = 0, 1, \cdots, 40\). There is an inclusion \(\Omega\) with pressure release surface occupies the region \(\{(x_1, x_2) \mid x_1^2 + (x_2 - 50)^2 \leq (50/\pi)^2\}\). If the waveguide is normalized to depth \(\pi\), then the normalized wave number \(k = 4\), which means there are four propagating modes for the acoustic wave at the given frequency.

We first generate the propagating wave by our approximate boundary integral equation method. More precisely, we solve the integral equation (2.14) for \(\psi(x, x^s)\) where \(p_0(x, x^s)\) is given by (2.7) with truncation at \(n = 30\) and \(x^s = (-500/\pi, 100/\pi)\), and substitute the \(\psi(x, x^s)\) into (2.18) to get the propagating field \(p(x, x^s)\). (A contour plotting of the propagating wave is plotted in figure 3). In particular, we obtain \(p_m^n = p(600/\pi, 2.5m; x^s), m = 0, 1, \cdots, 40\). To make these data more close to the reality, we add some Gaussian noise (generated by a NAG subroutine g05ddf in our computation) to this data and use them as our detected data.

The second step is to compute the estimator. Since there are only four propagating modes, we choose \(N = 10\) and compute \(B_n(x), n = 1, 2, \cdots, 10\). Using these \(B_n(x)\), we search the area of \([-900/\pi, -300/\pi] \times [0, 100]\), and plot the estimator \(F_p(x^s)\) for \(x^s \in [-900/\pi, -300/\pi] \times [0, 100]\). (See figures 4-10).

Figure 4-5: These figures show the estimator \(F_p(x^s)\) for the detected data \(p_m^* = p(600/\pi, 2.5m; x^s), m = 0, 1, \cdots, 40\) without adding Gaussian noise. Though these beautiful plottings have not too much sense in practice, we pose them here as theoretical expectancies and use them for comparison.

Figure 6-7: These figures show the estimator \(F_p(x^s)\) for the detected data \(p_m^* = p(600/\pi, 2.5m; x^s), m = 0, 1, \cdots, 40\) with about 10% Gaussian noise.

Figure 8-10: These figures show the estimator \(F_p(x^s)\) for the detected data \(p_m^* = p(600/\pi, 2.5m; x^s), m = 0, 1, \cdots, 40\) with about 100% Gaussian noise (the signal-noise ratio is about 1). In figure 8, we plot the contour of \(F_p(x^s)\) which shows that the signal is buried by the noise. In figure 9, a filter with the threshold value \(F_p(x^s) = 0.5\) is used, i.e. we set \(F_p(x^s) = 0\) if \(F_p(x^s) < 0.5\). In figure 10, the threshold value is increased to \(F_p(x^s) = 0.55\) and the source is clearly identified.

Example 2: Horizontal hydrophone array

The synthetic modeling of the computer simulation is depicted in figure 11.
We assume the waveguide, the inclusion and the other acoustic parameters are the same as that in example 1 except that the hydrophone array is arranged horizontally at \(((100j + 3000)/6\pi, 25/\pi), j = 0, 1, \cdots, 6\). In the same way as in example 1, we compute the estimator \(F_p(x^*)\) and plot it in figure 12-15.

**Figure 12-13:** These figures show the estimator \(F_p(x^*)\) for the detected data \(p_m^* = p((100j + 3000)/6\pi, 25/\pi), j = 0, 1, \cdots, 6\) without adding Gaussian noise.

**Figure 14-15:** These figures show the estimator \(F_p(x^*)\) for the detected data \(p_m^* = p((100j + 3000)/6\pi, 25/\pi), j = 0, 1, \cdots, 6\) with about 10% Gaussian noise.

\[ 4 \quad \text{Conclusion} \]

1. Matched field signal processing in complex environments are very interesting problems. One of the essential parts of these problems is to find an efficient and accurate algorithm to solve the propagating field. The scheme used here makes use of the separation of the source and the detection array, and greatly reduces the computation.

2. The signal processing method used here is a high resolution method. It localizes the source nicely even when a substantial amount of noise exists.

\[ 5 \quad \text{Appendix} \]

An approximate boundary integral equation method [13] is included in this Appendix for solving the boundary integral equation

\[ \psi(x) + 2 \int_{\partial \Omega} \frac{\partial p_0(x; y)}{\partial \nu_y} \psi(y) dy = 2f(x), \text{ for } x \in \partial \Omega. \quad (5.1) \]

Let

\[ G_0(x; y) := G_0(x_1, x_2; y_1, y_2) = \sum_{n=1}^{\infty} \frac{1}{\pi(n - \frac{1}{2})^2} \sin((n - \frac{1}{2})x_2) \sin((n - \frac{1}{2})y_2) e^{-(n-\frac{1}{2})|x_1-y_1|}, \quad (5.2) \]

and

\[ M(x; y) := p_0(x; y) - G_0(x; y) = \sum_{n=1}^{\infty} \frac{1}{\pi} \sin((n - \frac{1}{2})x_2) \sin((n - \frac{1}{2})y_2) \left( \frac{i}{ka_n} e^{i\alpha_n |x_1-y_1|} - \frac{1}{n-\frac{1}{2}} e^{-(n-\frac{1}{2})|x_1-y_1|} \right), \quad (5.3) \]

where \(a_n = [1 - (2n-1)^2/4k^2]^{\frac{1}{2}}\).

we can rewrite (5.1) in the form

\[ \psi(x) + 2 \int_{\partial \Omega} \frac{\partial G_0(x; y)}{\partial \nu_y} \psi(y) dy + 2 \int_{\partial \Omega} \frac{\partial M(x; y)}{\partial \nu_y} \psi(y) dy = 2f(x), \text{ for } x \in \partial \Omega. \quad (5.4) \]

We assume that the boundary \(\partial \Omega\) is given by a 2\(\pi\)-periodic parametric representation

\[ \gamma(s) = (\gamma_1(s), \gamma_2(s)), s \in \partial \Omega, \]

with \(|\gamma'(s)| \neq 0\) for all \(s\). Furthermore, we assume that \(\gamma\) is a \(C^\infty\) function.

Denote the kernel of the integral equation (5.1) by

\[ K_0(x; y) = 2 \frac{\partial}{\partial \nu_y} G(x; y), \quad K_1(x; y) = 2 \frac{\partial}{\partial \nu_y} M(x; y), \quad (5.5) \]
and set
\[ w(s) = ψ(γ(s)), \quad g(s) = 2f(γ(s)), \]
\[ L_0(s, σ) = K_0(γ(s); γ(σ))[γ′(σ)], \quad L_1(s, σ) = K_1(γ(s); γ(σ))[γ′(σ)]. \]

Thus equation (5.1) reduces to
\[ w(s) + \int_{-π}^{π} w(σ)L_0(s, σ)dσ + \int_{-π}^{π} w(σ)L_1(s, σ)dσ = g(s), \quad s ∈ [-π, π]. \tag{5.6} \]

It is shown [13] that \( L_0(s, σ) \) is continuous for \((s, σ) ∈ [-π, π] \times [-π, π],\) and that \( L_1(s, σ) \) can be written as
\[ L_1(s, σ) = -a(s, σ)\log|2\sin\frac{s - σ}{2}| + b(s, σ)\left(\arctan cot\frac{s + σ}{2} + \text{sgn}(s^2 - σ^2)\frac{π}{2}\right) + L_2(s, σ), \tag{5.7} \]
where \( a(s, σ), b(s, σ) \) and \( L_2(s, σ) \) are continuous and differentiable for \((s, σ) ∈ [-π, π] \times [-π, π].\)

We use the ordinary rectangular formula
\[ \int_{-π}^{π} v(σ)dσ ≈ h \sum_{k=-N/2+1}^{N/2} v(t_k), \tag{5.8} \]
the weighted quadrature formula
\[ -\int_{-π}^{π} v(σ)\log|2\sin\frac{s - σ}{2}|dσ ≈ h \sum_{k=-N/2+1}^{N/2} R^1(s - t_k)v(t_k), \tag{5.9} \]
and the weighted quadrature formula
\[ \int_{-π}^{π} v(σ)\left(\arctan cot\frac{s + σ}{2} + \text{sgn}(s^2 - σ^2)\frac{π}{2}\right)dσ ≈ h \sum_{k=-N/2+1}^{N/2} R^2(s, t_k)v(t_k), \tag{5.10} \]
where \( t_k = kh \) with \( h = \frac{2π}{N} \) and \( N \) an even integer are the equidistant quadrature knots and the weights are given by
\[ R^1(s) = \sum_{l=1}^{N/2-1} \frac{1}{l}\cos ls + \frac{1}{N}e^{i\frac{N}{2}s} \]
and
\[ R^2(s, t_k) = \sum_{l=-N/2+1}^{N/2} \left(\frac{\sin l|s|}{l} + \frac{ie^{-il}s}{2l}\right)e^{-ilt_k} + |s| - \frac{π}{2}. \]

Applying the quadrature formula (5.8), (5.9) and (5.10) to the integrals in (5.6), we replace the integral equation (5.6) by the linear system
\[ w_j + h \sum_{k=-N/2+1}^{N/2} (R_1(t_{j-k})a(t_j, t_k) + R_2(t_j, t_k)b(t_j, t_k) + L_0(t_j, t_k) + L_2(t_j, t_k))w_k = g_j, \tag{5.11} \]
\[ j = -\frac{N}{2} + 1, ..., \frac{N}{2}. \]

for the approximate values \( w_j \) to \( w(t_j), \) where \( g_j = g(t_j). \)

For more detail discussions, the reader could refer [13].
References


Figure 1: Acoustic source in a perturbed waveguide

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