BOUNDARY CONTROL OF A SCHROEDINGER EQUATION
WITH NONCONSTANT PRINCIPAL PART

By

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Boundary Control of a Schrödinger Equation with Nonconstant Principal Part

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Abstract
In this paper, we obtain a simple proof of exact boundary controllability for the Schrödinger equation with variable principal part by combining a method of W. Littman and S. Taylor with an estimate of B. Vainberg.

1 Introduction
In the context of boundary controllability for a Schrödinger equation, Littman and Taylor have introduced a general technique, which, basically stated, says that

\[\text{local smoothing } + \text{ reversibility } + \text{ uniqueness } \implies \text{exact controllability}\]

(see [4]). Here “uniqueness” is the uniqueness property implying approximate controllability by duality. If control is to be exercised on the whole lateral boundary of a cylindrical domain, \(\Omega \times (0, T)\), in space-time, it means that every solution (in an appropriate space) of the homogeneous linear evolution equation in this domain having zero Cauchy data on the lateral boundary must vanish identically. “Reversibility” means that the backward problem (in time) is wellposed. For the system we wish to consider, the difficulty of this approach lies in proving the necessary smoothing properties.

Local smoothing properties of evolution equations are not a newly discovered phenomenon. For example, the propagation of singularities for hyperbolic systems is well known. However, in [4], Littman and Taylor established similar properties for very different classes of equations, i.e., the Schrödinger equation with nonsmooth potentials, and that these smoothing properties can be applied in the method just described.

In our problem, equations exhibiting smoothing properties result in solutions belonging to the class Gevrey-\(\delta\), where this Gevrey class is defined with respect to the time variable \(t\). We formulate this more precisely below.

**Definition** A function \(f(x, t)\) belongs to the class Gevrey-\(\delta\) with respect to \(t\) and uniformly for \((x, t)\) in the compact set \(K\) if for all \((x, t) \in K\) and for all \(\theta > 0\), there exists a constant \(C_{K, \theta} > 0\) such that

\[
|\frac{\partial^n}{\partial t^n} f(x, t)| \leq C_{K, \theta} \theta^n (n!)^\delta.
\]

*This material is based upon work partially supported under a National Science Foundation Mathematical Sciences Postdoctoral Research Fellowship.*
With this definition in mind, we state Littman and Taylor’s result on local smoothing properties for a Schrödinger equation with potential in $\mathbb{R}^n$.

**Theorem 1.1** (See [4].) Consider the initial value problem in $\mathbb{R}^n$:

$$
\begin{align*}
    i\frac{\partial u}{\partial t} + \Delta u - V(x)u &= 0, & x \in \mathbb{R}^n, & t > 0 \\
    u(x, 0) &= \phi(x)
\end{align*}
$$

Assume that $V$ is bounded, measurable and has compact support and that $\phi(x) \in L^2(\mathbb{R}^n)$ and has compact support. Then the solution $u(x, t)$ is of class Gevrey-2 with respect to $t$ uniformly in compact sets of $\mathbb{R}^n \times \{t > 0\}$.

Extensions of the above result to the case of nonconstant coefficients in the principal part of the operator are of interest both mathematically and physically. On the other hand, techniques used in the proof of Theorem 1.1 take advantage of the structure of the Green’s function associated with the problem, which will no longer be possible in the more general case. We note that the proof in [4] was only given for $n = 3$ and that Taylor has extended the result to general $n$.

To begin our study of Schrödinger equations with nonconstant coefficients in the principal part of the differential operator, we consider the following system.

$$
\begin{align*}
    i\frac{\partial u}{\partial t} + Au &= 0, & x \in \mathbb{R}^n, & t > 0, \\
    u(x, 0) &= \phi(x),
\end{align*}
$$

(1.1)

where

$$
A \equiv \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i} a_{ij}(x) \frac{\partial}{\partial x_j} c(x).
$$

(1.2)

Note that, by definition, any potential is included in the operator $A$.

The question we wish to address is that of boundary controllability of the associated system,

$$
\begin{align*}
    i\frac{\partial u}{\partial t} + Au &= 0, & x \in \Omega, & t > 0, \\
    u(x, 0) &= g(x) \\
    u &= h(x, t) & x \in \partial\Omega, & t > 0,
\end{align*}
$$

(1.3)

where $\Omega \subset \mathbb{R}^n$ is a bounded domain. Given initial data in some appropriate space (e.g. $L^2(\mathbb{R}^n)$), can we find a function $h(x, t)$ such that for some $T > 0$, $u(x, t) = 0$ for all $t > T$?

Although various systems of equations have been considered and a variety of techniques developed in the context of boundary control, little has been done specifically for the Schrödinger equation. Hence, the following references are of particular interest. In [2], Lasiecka and Triggiani use the method of multipliers to obtain boundary controllability for the Schrödinger equation with constant coefficients. In [3], Lebeau modifies the results of Bardos, Lebeau and Rauch (see [1]) on exact controllability for hyperbolic equations to apply their techniques to Schrödinger type equations. However, in the statement of his results, Lebeau assumes constant coefficients (i.e., $A$ replaced by $\Delta$) and an analytic boundary. Although his work appears to extend to the case of nonconstant coefficients, our goal is to avoid the technical difficulties of applying his techniques to (1.3).

To instead take advantage of Littman and Taylor’s technique, we prove the following regularity result.
Theorem 1.2 Assume the coefficients of \( A \) are real and satisfy the following conditions:

\[
\begin{align*}
&i.) \quad a_{ij}(x) = a_{ji}(x) \quad \text{for all } i, j, \\
&ii.) \quad \sum_{i,j=1}^n a_{ij}(x) \xi_i \xi_j \geq \alpha |\xi|^2 \quad \text{for some } \alpha > 0, \\
&iii.) \quad a_{ij}(x) - \delta_{ij}, c(x) \in C_0^\infty(\mathbb{R}^n),
\end{align*}
\]  

(1.4)

where \( \delta_{ij} \) is the Kronecker delta. Without loss of generality, we assume that the support of these functions is contained in the ball \( B_a \equiv \{ x : |x| < a \} \).

Assume \( \phi(x) \in L^2(\mathbb{R}^n) \) and has compact support. Then the solution \( u(x,t) \) to the initial value problem (1.1) is of class Gevrey-2 with respect to \( t \) uniformly in compact sets of \( \mathbb{R}^n \times \{ t > 0 \} \).

For the convenience of the reader, in the proof of this theorem, we reproduce some of the arguments and computations from [4]. Extensions to nonsmooth coefficients will appear elsewhere.

It should be noted that using Littman and Taylor’s technique to prove exact controllability results in the control function belonging to \( C^\infty(\partial \Omega) \) for \( t > 0 \). As far as Sobolev spaces are concerned, the results are the same as in [4], i.e., for initial data in \( L^2(\Omega) \), the Dirichlet controls will also be in \( H^1(\partial \Omega) \), indeed, somewhat better.

2 Gevrey Regularity of the Solution: Proof of Theorem 1.2

**Step 1: Laplace Transform.** Assuming the coefficients satisfy (1.4), the operator \( A \) is strongly elliptic and self-adjoint. Additionally, we can apply semigroup theory (see [5]) to find that \( iA \) is the infinitesimal generator of a strongly continuous group of unitary operators. Taking the Laplace transform of (1.1) with respect to the time variable, \( t \), we find

\[
i\lambda w + Aw = i\phi
\]  

(2.1)

or

\[
k^2 w + Aw = i\phi,
\]  

(2.2)

where \( \lambda = -ik^2 \). Thus, the solution operator mapping \( \phi \to w \) is the resolvent operator corresponding to \( iA \), where the resolvent is defined to be

\[
R(\lambda; iA) \equiv (\lambda - iA)^{-1}.
\]  

(2.3)

Since \( iA \) generates a group of unitary operators, (2.1) is solvable for any \( \phi \in L^2(\mathbb{R}^n) \) for all \( \lambda \) such that \( \Re \lambda > 0 \) and the resulting solution, \( w \), lies in \( H^2(\mathbb{R}^n) \).

**Step 2: Estimates for the Modified Resolvent.** To prove the desired estimates for the solution which are needed to show that it is of class Gevrey-2 with respect to time, we must extend the region where (2.1) is solvable, i.e., the region where the resolvent exists and is bounded. However, this cannot be done directly. Instead, we consider a “modified” resolvent operator,

\[
R_x(\lambda; iA) \equiv \chi(x)R(\lambda; iA)\chi(x),
\]  

(2.4)

where \( \chi(x) \in C_0^\infty(\mathbb{R}^n) \) and \( \chi(x) \equiv 1 \) on \( \text{supp}\phi \).

Recalling the change of variable, \( \lambda = -ik^2 \),

\[
\begin{align*}
\lambda - iA &= -i(k^2 + A) \\
\Rightarrow R(\lambda; iA) &= i(k^2 + A)^{-1}.
\end{align*}
\]  

(2.5)
Hence, we will consider the operator,

$$\tilde{R}(k^2; \mathcal{A}) \equiv \chi(x)(k^2 + \mathcal{A})^{-1}\chi(x). \quad (2.6)$$

Notice that the interior term differs from the standard definition of a resolvent operator by only a change of sign. For this operator, Vainberg has shown ([6], Theorem 3) that if

$$U_{\alpha, \beta} \equiv \{ k : \alpha|k|^{-1}\exp\{\beta|\Im mk|\} \leq \frac{1}{2}\}, \quad (2.7)$$

then $\tilde{R}(k^2; \mathcal{A})$ has no poles in $U_{\alpha, \beta}$ and

$$\|\tilde{R}(k^2; \mathcal{A})\| \leq c_1|k|^{-1}\exp\{c_2|\Im mk|\}, \quad k \in U_{\alpha, \beta}. \quad (2.8)$$

In particular, this implies

$$\|\tilde{R}(k^2; \mathcal{A})\| \leq C, \quad k \in U_{\alpha, \beta}. \quad (2.9)$$

**Step 3: Transformation from $k$ to $\lambda$.** Because the results of Step 2 are in terms of the variable $k$, we need to convert the information derived from Vainberg’s work into estimates in terms of $\lambda$ to obtain corresponding results for $R_\chi(\lambda; i\mathcal{A})$. We begin by transforming the region in which estimate (2.9) is valid. Let

$$S_{a, b, R} \equiv \{ k : |k| > R, \Im mk \geq -a \ln |k| + b \}. \quad (2.10)$$

By appropriately choosing $a$, $b$, and $R$, it is a straightforward calculation to show that $S_{a, b, R} \subset U_{\alpha, \beta}$. Restricting the region further, if

$$T_{a', b'} \equiv \{ k : \Im mk \geq -b', |k| \geq a' \}, \quad (2.11)$$

we can choose $a'$ and $b'$ so that $T_{a', b'} \subset S_{a, b, R}$.

Since the transformation $\lambda = -ik^2$ is two to one on parts of the set $T_{a', b'}$, we consider the restriction of this set,

$$\chi_{a, b} \equiv T_{a, b} \cap \{ k : -\frac{\pi}{4} < \arg k < \frac{3\pi}{4} \}. \quad (2.12)$$

We now apply the following lemma.

**Lemma 2.1 ([4], Lemma 3)** *For each $a > 0$ there exists a constant $\rho > 0$ such that the set

$$\Sigma_{b, \rho} \equiv \{ \lambda : \Re \lambda \geq -b|\Im \lambda|^{1/2} + \rho \}$$

is contained in $\chi_{a, b}$.*

Since the estimate (2.9) is uniform in $k$, it is also valid for all $\lambda \in \Sigma_{b, \rho}$. Therefore,

$$\|R_\chi(\lambda; i\mathcal{A})\| \leq C \quad \forall \lambda \in \Sigma_{b, \rho}. \quad (2.13)$$

**Step 4: Estimates for the Solution.** Define

$$J(t) \equiv \frac{1}{2\pi i} \int_{\mu-i\infty}^{\mu+i\infty} \frac{\tilde{R}(\lambda; i\mathcal{A})}{\lambda^2} e^{\lambda t} d\lambda, \quad (2.14)$$

where $\mu$ is chosen sufficiently large to guarantee that the half-plane $\Re \lambda \geq \mu$ is contained in the domain of analyticity of $J$. 


Theorem 2.1 Let $K$ be any compact subset of $\mathbb{R}^n$. Then the mapping $t \to J(t)$ is of class Gevrey-2 with respect to time.

Proof: Step 1: To derive estimates for $J(t)$, we begin by proving that the path of integration can be shifted to the contour $\Gamma$, where $\Gamma \equiv \partial \Sigma_{\lambda, \rho}$ with orientation in the direction of increasing $\Im \lambda$.

Consider the contour $\Gamma_R \equiv \{ \lambda = s + iR : a - b|R|^{1/2} \leq s \leq \mu \}$ with orientation in the direction of increasing $\Re \lambda$. Then
\[
| \int_{\Gamma_R} \frac{\hat{R}(\lambda; iA)}{\lambda^2} e^{\lambda t} d\lambda | \leq \frac{1}{|R|^2} \int_{|a-b|R|^{1/2}}^\mu C e^{\xi t} d\xi 
\leq \frac{C}{|R|^2} \frac{e^{\xi t}}{\xi} \to 0 \quad \text{as} \quad |R| \to \infty \quad \forall t > 0. \tag{2.15}
\]

Thus, the path of integration can be shifted to the contour $\Gamma$.

Step 2: Estimates for Derivatives. As in [4], we parametrize the upper branch of the contour (i.e., $\Im \lambda \geq 0$) in the following way:
\[
\lambda = -\mu + i\left(\frac{\rho + \mu}{b}\right)^2. \tag{2.16}
\]

Then, since the integral is infinitely differentiable, derivatives of $J(t)$ may be bounded in the following way, using (2.9) and the technique of the proof of Theorem 4 in [4]:
\[
| \frac{\partial^n}{\partial t^n} \int_\Gamma \frac{\hat{R}(\lambda; iA)}{\lambda^2} e^{\lambda t} d\lambda | = | \int_\Gamma \lambda^{n-2} \hat{R}(\lambda; iA) e^{\lambda t} d\lambda |
\leq C \int_0^\infty (\mu^2 + (\frac{\rho + \mu}{b})^4)^{(n-2)/2} (1 + 4(\frac{\rho + \mu}{b})^2)^{1/2} e^{-\mu t} d\mu
\leq C 2^{(n+1)/2} \int_0^\infty (\frac{\rho + \mu}{b})^{2n-3} e^{-\mu t} d\mu \quad \text{since} \quad \rho > b^2
\leq C b e^\rho t 2(n+1)/2 \int_0^\infty (\frac{\rho + \mu}{b})^{2n-3} e^{-\mu t} d\mu
= 2C b e^\rho t (2n - 3)! \left(\frac{\rho}{b}\right)^{(n-1)/2}. \tag{2.17}
\]

If $t$ is in any compact subset of $\mathbb{R}^+$ and $\theta > 0$, we may choose $b$ so that this expression is bounded by $c\theta^n (n!)^2$, where $c$ is constant. \square

Theorem 2.2 Assume the coefficients of $A$ satisfy (1.4). Let $K$ be any compact subset of $\text{supp} \phi$. If $\phi(x) \in L^2(\mathbb{R}^n)$ has compact support, then the mapping $t \to u(\cdot, t)$ is of class Gevrey-2 with respect to time.

Proof: By a standard semigroup result (see [5], Corollary 1.7.6),
\[
\int_0^t (t-s)S(s)\phi ds = \frac{1}{2\pi i} \int_{\beta-i\infty}^{\beta+i\infty} \frac{R(\lambda; iA)}{\lambda^2} \phi e^{\lambda t} d\lambda, \tag{2.18}
\]
where $R(\lambda; iA)$ is the resolvent of the infinitesimal generator of our group $S(t)$, and $\beta$ is assumed to be sufficiently large. However, since $\chi(x) \equiv 1$ on $\text{supp} \phi$, the above equation implies
\[
\chi \int_0^t (t-s)S(s)\chi \phi ds = J(t)\phi. \tag{2.19}
\]
Differentiating twice with respect to $t$ then gives
\[
\chi(x)u(x, t) = \frac{\partial^2}{\partial t^2} J(t)\phi. \tag{2.20}
\]
Hence, since $J(t)$ is of class Gevrey-2 and differentiation with respect to $t$ does not affect Gevrey regularity, $\chi(x)u(x, t)$ is also of class Gevrey-2 with respect to time. \square
References


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