ON THE PERTURBATION OF MARKOV CHAINS
WITH NEARLY TRANSIENT STATES

By

G.W. Stewart

IMA Preprint Series # 953
April 1992
On the Perturbation of Markov Chains withNearly Transient States*

G. W. Stewart†

ABSTRACT

Let $A$ be an irreducible stochastic matrix of the form

$$A = \begin{pmatrix} A_{11} & E_{12} \\ A_{21} & A_{22} \end{pmatrix}.$$ 

If $E_{22}$ were zero, the states corresponding to $A_{22}$ would be transient in the sense that if the steady state vector $y^T$ is partitioned conformally in the form $(y_1^T \ y_2^T)$ then $y_2^T = 0$. If $E_{22}$ is small, then $y_2^T$ will be small, and the states are said to be nearly transient. It this paper it is shown that small relative perturbations in $A_{11}$, $A_{21}$, and $A_{22}$, though potentially larger than $y_2^T$, induce only small relative perturbations in $y_2^T$.

---

*This report is available by anonymous ftp from thales.cs.umd.edu in the directory pub/reports.

†Department of Computer Science and Institute for Advanced Computer Studies, University of Maryland, College Park, MD 20742. This work was supported in part by the Air Force Office of Scientific Research under Contract AFOSR-87-0188 and was done while the author was a visiting faculty member at the Institute for Mathematics and Its Applications, The University of Minnesota, Minneapolis, MN 55455.
On the Perturbation of Markov Chains with Nearly Transient States

G. W. Stewart

ABSTRACT

Let $A$ be an irreducible stochastic matrix of the form

$$A = \begin{pmatrix}
A_{11} & E_{12} \\
A_{21} & A_{22}
\end{pmatrix}.$$

If $E_{22}$ were zero, the states corresponding to $A_{22}$ would be transient in the sense that if the steady state vector $y^T$ is partitioned conformally in the form $(y_1^T \quad y_2^T)$ then $y_2^T = 0$. If $E_{22}$ is small, then $y_2^T$ will be small, and the states are said to be nearly transient. It this paper it is shown that small relative perturbations in $A_{11}$, $A_{21}$, and $A_{22}$, though potentially larger than $y_2^T$, induce only small relative perturbations in $y_2^T$.

1. Introduction

The concerns of the paper are best illustrated by a $2 \times 2$ example. Consider the nonnegative matrix

$$A_2 = \begin{pmatrix}
1 - \epsilon & \epsilon \\
\alpha & 1 - \alpha
\end{pmatrix},$$

where $\epsilon$ is small and $\alpha$ is of order of magnitude one (e.g., $\alpha = \frac{1}{2}$). When $\epsilon = 0$, the second state of the Markov chain corresponding to $A_2$ is transient: it eventually goes away, never to return. When $\epsilon$ is small but positive, we shall say that the state is nearly transient. The near transience of the state is reflected by the second component of the steady state vector

$$y^T = \frac{1}{1 + \epsilon/\alpha} \left( \frac{\epsilon}{\alpha} \right),$$

which tends to zero with $\epsilon$.

Now consider the perturbed matrix

$$\tilde{A}_2 = \begin{pmatrix}
1 - \epsilon - \eta & \epsilon \\
\alpha & 1 - \alpha
\end{pmatrix},$$
where $\eta$ is small compared to one, but not necessarily small compared to $\epsilon$. The characteristic equation of $I - A_2$ is
\[ \lambda^2 - (\alpha + \epsilon + \eta)\lambda + \alpha \eta = 0. \]

By the quadratic formula, twice the smallest eigenvalue $\lambda$ of $I - A_2$ is given by
\[
2\lambda = \alpha + \epsilon + \eta - \sqrt{(\alpha + \epsilon + \eta)^2 - 4\alpha \eta} \\
\cong \alpha + \epsilon + \eta - \sqrt{\alpha^2 + 2\alpha (\epsilon + \eta) - 4\alpha \eta} \\
= \alpha + \epsilon + \eta - \alpha \sqrt{1 + 2(\epsilon - \eta)/\alpha} \\
\cong \alpha + \epsilon + \eta - \alpha (1 + (\epsilon - \eta)/\alpha) \\
= 2\eta. \tag{1.1}
\]

Here we have ignored terms of order $\eta^2$, as indicated by the symbol $\cong$.

If we seek the eigenvector corresponding to $\lambda$ in the form $\tilde{y}^T = (1 \ \xi)$, then from the second component of the equation $\tilde{y}^T(I - A_2) = \eta \tilde{y}^T + O(\eta^2)$ it follows that
\[-\epsilon + \xi \alpha = \eta \xi + O(\eta^2) \xi,
\]
which after some manipulation can be written in the form
\[ \xi = \frac{\epsilon}{\alpha} (1 + O(\eta)). \]

In other words, a change of order $\eta$ in the leading element of $A_2$ makes a relative change of only $O(\eta)$ in the components of the steady-state vector. This implies that the probability of being in the nearly transient state, however small, is insensitive to potentially much larger perturbations in the $(1,1)$-element of the matrix.

The purpose of this paper is to generalize this result to stochastic matrices of the form
\[ A = \begin{pmatrix} A_{11} & E_{12} \\ A_{21} & A_{22} \end{pmatrix}, \tag{1.2} \]
that is, to a chain with a group of nearly transient states. Before proceeding, however, it will be worth while to examine the above example more closely for things to generalize.

It is easy to see that the steady-state vector is also insensitive to small perturbations in the $(2,1)$- and $(2,2)$-elements of $A_2$. We will show that this generalizes: under suitable restrictions on $A$, the small components of the steady-state
vector corresponding to the nearly transient states are insensitive to perturbations in $A_{11}$, $A_{12}$, and $A_{22}$. On the other hand, in the example the small component of the steady-state vector is very sensitive to changes in $\epsilon$ itself, and we may expect a similar sensitivity in the general case to perturbations in $E_{12}$.

The condition that $\alpha = O(1)$ is necessary. For as as $\alpha$ becomes small, the approximations in (1.1) become increasingly inaccurate and break down entirely when $\alpha = O(\epsilon)$. (This break-down agrees with what we know about the perturbation theory for nearly completely decomposable chains, where the steady state vector is sensitive to such perturbations [2].) In generalizing the result, however, it is not be enough to require that $A_{21} = O(1)$, and we will formulate an alternate condition in terms of $A_{22}$.

Finally, we note that the perturbation in the example does not leave the matrix stochastic. At first blush, this generality may seem superfluous, since in applications to Markov chains we should expect both the matrix and its perturbation to be stochastic. However, it turns out that certain numerical algorithms, among them Gaussian elimination, introduce perturbations that render the matrix in question nonstochastic.

The paper is organized as follows. In the next section, we will introduce some preliminary transformations of the problem. In Section 3 we will establish a general perturbation bound, and in Section 4 we will discuss its consequences.

Throughout this paper $\| \cdot \|$ stands for the Euclidean vector norm and the subordinate matrix norm defined by

$$\|A\| = \sup_{\|x\|=1} \|Ax\|.$$  

2. The Transformed Problem

To state our problem more precisely, let the matrix $A$ of (1.2) and its submatrices $A_{11}$ and $A_{22}$ be irreducible, and let

$$y = (y_1^T \quad y_2^T)$$

be its Perron vector partitioned conformally. Let

$$G = \begin{pmatrix} G_{11} & 0 \\ G_{21} & G_{22} \end{pmatrix}$$
be a matrix that is small compared to one (but not necessarily compared to $E_{11}$), and assume that

$$\tilde{A} \equiv A + G = \begin{pmatrix} A_{11} + G_{11} & E_{12} \\ A_{21} + G_{21} & A_{22} + G_{22} \end{pmatrix}$$

is also irreducible. Let

$$\tilde{y} = (\tilde{y}_1^T \tilde{y}_2^T)$$

be the Perron vector of $\tilde{A}$. Then our problem is to establish perturbation bounds for $\tilde{y}_2^T$.

A technical difficulty presents itself immediately. If $E_{12}$ is small, the matrix $A_{11}$ is near a stochastic matrix and has an eigenvalue near one. Hence $I - A_{11}$ is very nearly singular, and this near singularity prevents us from applying standard perturbation theory directly. We will circumvent the problem by transforming the matrix $A$ into a form in which the offending eigenvalue is isolated.

Let $\beta_{11}$ be the Perron eigenvalue of $A_{11}$ and let the corresponding positive left eigenvector be $u_1^T$, normalized so that $\|u_1\| = 1$. Let

$$U = (u_1 \ U_2)$$

be orthogonal. Then it is easily verified that $U^T A_{11}$ has the form

$$U^T A_{11} U = \begin{pmatrix} \beta_{11} & 0 \\ b_{21} & B_{22} \end{pmatrix}.$$ 

The eigenvalues of $B_{22}$ are the eigenvalues of $A_{11}$ other than $\beta_{11}$. Since $A_{11}$ is substochastic, $I - B_{22}$ is nonsingular.

Now let

$$\begin{pmatrix} u_1^T \\ U_2 \end{pmatrix} E_{12} = \begin{pmatrix} f_{13} \\ F_{23} \end{pmatrix}$$

and

$$A_{21} (u_1 \ U_2) = (b_{31} \ B_{32}).$$

Then

$$B \equiv \begin{pmatrix} U^T \\ 0 \end{pmatrix} \begin{pmatrix} A_{11} & E_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} U \\ 0 \end{pmatrix} = \begin{pmatrix} \beta_{11} & 0 & f_{13}^T \\ b_{21} & B_{22} & F_{23} \\ b_{31} & B_{32} & B_{33} \end{pmatrix},$$

where $B_{33} = A_{22}$. 

Nearly Transient Chains
Nearly Transient Chains

Since both \( y_1^T \) and \( u_1^T \) are positive, \( y_1^T u_1 > 0 \). It follows that we may renormalize \( y^T \) so that

\[
y^T U = (1 \quad p_2^T \quad p_3^T),
\]

where \( p_2^T = y_1^T U_2 \) and \( p_3^T = y_2^T \). In terms of the transformed problem, our goal is to find perturbation bounds on \( p_3^T \), when the quantities \( B_{ij} \) are subject to perturbations.

It is easy to obtain a linear equation for \( p_3^T \). Because \( y^T \text{diag}(U, I) \) is a null vector of \( I - B \) it follows that \( p_2^T \) and \( p_3^T \) satisfy

\[
(p_2^T \quad p_3^T) \begin{pmatrix}
I - B_{22} & -F_{23} \\
-B_{32} & I - B_{33}
\end{pmatrix} = (0 \quad f_{13}^T).
\]

Eliminating \( p_2^T \) from this equation, we obtain

\[
p_3^T \left( I - B_{33} - B_{32}(I - B_{22})^{-1}F_{23} \right) = f_{12}^T.
\]

(2.1)

It is equally easy to obtain an equation for the perturbed vector \( \tilde{p}_3 \). Let

\[
H = U^T G U = \begin{pmatrix}
\eta_{11} & h_{12}^T & 0 \\
h_{21} & H_{22} & 0 \\
h_{31} & H_{32} & H_{33}
\end{pmatrix},
\]

and assume that \( \tilde{A} = A + G \) is stochastic. Then in the transformed system (with tildes denoting the obvious perturbations)

\[
(p_{\tilde{2}}^T \quad p_{\tilde{3}}^T) \begin{pmatrix}
I - \tilde{B}_{22} & -\tilde{F}_{23} \\
-\tilde{B}_{32} & I - \tilde{B}_{33}
\end{pmatrix} = (h_{12}^T \quad f_{13}^T).
\]

It follows that

\[
p_{\tilde{3}}^T \left( I - \tilde{B}_{33} - \tilde{B}_{32}(I - \tilde{B}_{22})^{-1}\tilde{F}_{23} \right) = f_{12}^T - h_{12}^T(I - \tilde{B}_{22})^{-1}\tilde{F}_{23}.
\]

(2.2)

3. The Perturbation Bound

In this section we will establish perturbation bounds for \( \tilde{p}_3 \). It will be convenient to have an abbreviated notation for the norms occurring in the bounds. Accordingly, we set

\[
\beta \equiv \|B\| = \|A\|
\]

\[
\eta \equiv \|H\| = \|G\|
\]

\[
\gamma_i \equiv \|(I - A_{ii})\|, \quad (i = 2, 3).
\]

(3.1)
The equalities in the above definitions follow from the fact that a transformation by the orthogonal matrix \( U \) does not change the spectral norm. The same symbols with tildes denote the norms of the perturbed quantities; e.g., \( \tilde{\beta} = \| \tilde{B} \| \).

We begin by collecting some standard results from the perturbation of linear systems (see, e.g., [1,3]).

**Theorem 3.1.** Let \( C \) be nonsingular and let \( \tilde{C} = C + Q \), where

\[
\| C^{-1} \| \| Q \| < 1.
\]

Then \( \tilde{C} \) is nonsingular,

\[
\| \tilde{C}^{-1} \| \leq \frac{\| C^{-1} \|}{1 - \| C^{-1} \| \| Q \|}, \tag{3.2}
\]

and

\[
\| \tilde{C}^{-1} - C^{-1} \| \leq \frac{\| C^{-1} \| \| Q \|}{1 - \| C^{-1} \| \| Q \|}, \tag{3.3}
\]

Moreover, if

\[
x^T C = d^T \quad \text{and} \quad \tilde{x}^T \tilde{C} = d^T + q^T,
\]

then

\[
\frac{\| \tilde{x} - x \|}{\| x \|} = \frac{\| C^{-1} \|}{1 - \| C^{-1} \| \| Q \|} \left( \| Q \| + \frac{\| q \|}{\| x \|} \right). \tag{3.4}
\]

Now let \( C \) denote the matrix in equation (2.1) for \( p_3^T \) and let \( d^T \) denote the right-hand side. Let \( \tilde{C} \) denote the matrix in the perturbed system (2.2) and \( \tilde{d}^T \) denote the right-hand side. To apply Theorem 3.1, we must bound \( \| \tilde{C} - C \| \) and \( \| \tilde{d}^T - d^T \| \).

We have

\[
\tilde{C} - C = (B_{33} - \tilde{B}_{33}) + (B_{32} - \tilde{B}_{32})(I - \tilde{B}_{22})^{-1} F_{23} + B_{32} \left( (I - B_{22})^{-1} - (I - \tilde{B}_{22})^{-1} \right) F_{23}.
\]

On taking norms we get

\[
\| \tilde{C} - C \| \leq \eta + \eta \tilde{\gamma}_2 \epsilon + \beta \frac{\eta \epsilon \tilde{\gamma}_2}{1 - \gamma_2 \eta},
\]

The third term of the bound follows from (3.3) under the assumption that \( \eta \gamma_2 < 1 \). Since from (3.2) we have \( \tilde{\gamma}_2 \leq \gamma_2/(1 - \eta \gamma_2) \), if we set \( \tilde{\eta} = \eta/(1 - \eta \gamma_2) \), we have

\[
\| \tilde{C} - C \| \leq \tilde{\eta}(1 + \gamma \epsilon + \beta \gamma \epsilon).
\]
Similarly,

\[ \| \tilde{d} - d\| \leq \tilde{\eta} \epsilon. \]

If we now use these bounds in (3.4), we get the following theorem (remember that \( y^T_2 = p^T_3 \)).

**Theorem 3.2.** Let the irreducible stochastic matrix \( A \) have the form

\[ A = \begin{pmatrix} A_{11} & E_{12} \\ A_{21} & A_{22} \end{pmatrix}, \]

where \( A_{11} \) and \( A_{22} \) are irreducible, and let

\[ \tilde{A} = A + G \equiv \begin{pmatrix} A_{11} & E_{12} \\ A_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} G_{11} & 0 \\ G_{21} & G_{22} \end{pmatrix} \]

also be stochastic. In the notation of (3.1), assume that \( \eta \gamma_2 < 1 \), and set

\[ \tilde{\eta} = \frac{\eta}{1 - \eta \gamma_2} \quad \text{and} \quad \mu = 1 + \gamma \epsilon + \beta \gamma \epsilon. \]

If

\[ \tilde{\eta} \mu \gamma_3 < 1, \]

then

\[ \frac{\| \tilde{y}^T_2 - y^T_2 \|}{\| y^T_2 \|} \leq \frac{\tilde{\eta} \gamma_3}{1 - \tilde{\eta} \mu \gamma_3} \left[ \mu + \frac{\epsilon}{\| y^T_2 \|} \right]. \quad (3.5) \]

**4. Discussion**

The bound (3.5) gives the promised result. Provided \( \| y^T_2 \| \) is of order \( \epsilon \) (more on this point later), the relative perturbation of \( y^T_2 \) is a small multiple of \( \eta \). The condition that \( \tilde{\eta} \mu \gamma_3 < 1 \) is the condition on \( A_{22} \) mentioned in the introduction. It essentially says that the eigenvalues of \( A_{22} \) are bounded away from one. In particular, it prevents the matrix \( A_{21} \) from being small—the condition used in the 2 × 2 example in the introduction.

It is instructive to examine the asymptotic form of the bound as \( \epsilon \) and \( \eta \) approach zero. In this case, \( \mu \) approaches one and \( \tilde{\eta} \) approaches \( \eta \). Consequently, (3.5) has the asymptotic form

\[ \frac{\| \tilde{y}^T_2 - y^T_2 \|}{\| y^T_2 \|} \lesssim \eta \gamma_3 \left[ 1 + \frac{\epsilon}{\| y^T_2 \|} \right]. \quad (4.1) \]
Thus if $\epsilon/\|y_2^T\|$ is near one, the factor controlling the size of the perturbation is $\gamma_3$; i.e., the norm of $(I - A_{33})^{-1}$.

The requirement that $\epsilon/\|y_2^T\|$ be near one may seem awkward, but it is necessary. If $y_2^T$ is smaller than $\epsilon$, perturbations due to the interaction of $G$ and $E$ can obliterate it [see the right-hand side of (2.2)]. More insight into this phenomena can be gained by replacing $\|y_2^T\|$ by a lower bound. Since $p_3^T(I - B_{33}) = f_{13}^T$, it follows that $\|p_3\| \leq \|f_{13}^T\|/\|I - B_{33}\|$. Hence another, weaker asymptotic bound is

$$\frac{\|\tilde{y}_2^T - y_2^T\|}{\|y_2^T\|} \lesssim \eta \gamma_3 \left[ 1 + (1 + \beta) \frac{\epsilon}{\|f_{13}^T\|} \right].$$

(4.2)

Since $\beta$ is of order one, we see that the bound can become large when $f_{13}$ to be small compared with the matrix $E_{12}$.

Finally, we return to the case where $\tilde{A}$ is not stochastic. The problem here is that we have assumed the existence of a null vector for $I - \tilde{A}$ in deriving (2.2). We will circumvent this problem by perturbing $\tilde{A}$ so that $I - \tilde{A}$ is singular.

First note that from Theorem 3.1 and (2.1) we have the following bound:

$$\|y_2^T\| \leq \frac{\gamma_3 \epsilon}{1 - \beta \gamma_2 \epsilon};$$

i.e., the near transient states have probability of order $\epsilon$. Since one is a simple eigenvector of $A$, for $\eta$ sufficiently small there is a corresponding eigenvalue of $\tilde{A}$ of the form

$$\lambda = 1 + (y_1^T y_2^T) \begin{pmatrix} G_{11} & 0 \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} e \\ e \end{pmatrix} + O(\eta^2)$$

$$= 1 + y_1^T G_{11} e + O(\eta^2) + O(\epsilon)$$

(see [3, Theorem IV.2.3]). Hence $\|1 - \lambda\| \leq \|y_1\| \eta + O(\eta^2) + O(\epsilon)$. Thus if $\tilde{A} = \tilde{A} = (1 - \lambda)I$, then $\tilde{A}$ comes from a perturbation of $A$ whose norm is asymptotically bounded by $\eta(1 + \|y_1^T\|)$. Moreover, $I - \tilde{A}$ is exactly singular. Consequently, the asymptotic bounds (4.1) and (4.2) continue to hold with $\eta$ replaced by $\eta(1 + \|y_1^T\|)$.

References


<table>
<thead>
<tr>
<th>#</th>
<th>Author/s</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>866</td>
<td>Gui-Qiang Chen and Tai-Ping Liu</td>
<td>Zero relaxation and dissipation limits for hyperbolic conservation laws</td>
</tr>
<tr>
<td>867</td>
<td>Gui-Qiang Chen and Jian-Guo Liu</td>
<td>Convergence of second-order schemes for isentropic gas dynamics</td>
</tr>
<tr>
<td>868</td>
<td>Aleksander M. Simon and Zbigniew J. Grzywna</td>
<td>On the Larché-Cahn theory for stress-induced diffusion</td>
</tr>
<tr>
<td>869</td>
<td>Jerzy Luczka, Adam Gadomski and Zbigniew J. Grzywna</td>
<td>Growth driven by diffusion</td>
</tr>
<tr>
<td>870</td>
<td>Mitchell Luskin and Tsong-Whay Pan</td>
<td>Nonplanar shear flows for nonaligning nematic liquid crystals</td>
</tr>
<tr>
<td>871</td>
<td>Mahmoud Affouf</td>
<td>Unique global solutions of initial-boundary value problems for thermodynamic phase transitions</td>
</tr>
<tr>
<td>872</td>
<td>Richard A. Brualdi and Keith L. Chavez</td>
<td>Rectangular L-matrices</td>
</tr>
<tr>
<td>873</td>
<td>Xinfu Chen, Avner Friedman and Bei Hu</td>
<td>The thermistor problem with zero-one conductivity II</td>
</tr>
<tr>
<td>874</td>
<td>Raoul LePage</td>
<td>Controlling a diffusion toward a large goal and the Kelly principle</td>
</tr>
<tr>
<td>875</td>
<td>Raoul LePage</td>
<td>Controlling for optimum growth with time dependent returns</td>
</tr>
<tr>
<td>876</td>
<td>Marc Hallin and Madan L. Puri</td>
<td>Rank tests for time series analysis a survey</td>
</tr>
<tr>
<td>877</td>
<td>V.A. Solonnikov</td>
<td>Solvability of an evolution problem of thermocapillary convection in an infinite time interval</td>
</tr>
<tr>
<td>878</td>
<td>Horia I. Ene and Bogdan Vernescu</td>
<td>Viscosity dependent behaviour of viscoelastic porous media</td>
</tr>
<tr>
<td>879</td>
<td>Kaushik Bhattacharya</td>
<td>Self-accommodation in martensite</td>
</tr>
<tr>
<td>880</td>
<td>D. Lewis, T. Ratiu, J.C. Simo and J.E. Marsden</td>
<td>The heavy top: a geometric treatment</td>
</tr>
<tr>
<td>881</td>
<td>Leonid V. Kalachev</td>
<td>Some applications of asymptotic methods in semiconductor device modeling</td>
</tr>
<tr>
<td>882</td>
<td>David C. Dobson</td>
<td>Phase reconstruction via nonlinear least-squares</td>
</tr>
<tr>
<td>883</td>
<td>Patricio Aviles and Yoshikazu Giga</td>
<td>Minimal currents, geodesics and relaxation of variational integrals on mappings of bounded variation</td>
</tr>
<tr>
<td>884</td>
<td>Patricio Aviles and Yoshikazu Giga</td>
<td>Partial regularity of least gradient mappings</td>
</tr>
<tr>
<td>885</td>
<td>Charles R. Johnson and Michael Lundquist</td>
<td>Operator matrices with chordal inverse patterns</td>
</tr>
<tr>
<td>886</td>
<td>B.J. Bayly</td>
<td>Infinitely conducting dynamos and other horrible eigenproblems</td>
</tr>
<tr>
<td>887</td>
<td>Charles M. Elliott and Stefan Luckhaus</td>
<td>'A generalised diffusion equation for phase separation of a multi-component mixture with interfacial free energy'</td>
</tr>
<tr>
<td>888</td>
<td>Christian Schmeiser and Andreas Unterreiter</td>
<td>The derivation of analytic device models by asymptotic methods</td>
</tr>
<tr>
<td>889</td>
<td>LeRoy B. Beasley and Norman J. Pullman</td>
<td>Linear operators that strongly preserve the index of imprimitivity</td>
</tr>
<tr>
<td>890</td>
<td>Jerry Donato</td>
<td>The Boltzmann equation with lie and cartan</td>
</tr>
<tr>
<td>891</td>
<td>Thomas R. Hoffend Jr., Peter Smereka and Roger J. Anderson</td>
<td>A method for resolving the laser induced local heating of moving magneto-optical recording media</td>
</tr>
<tr>
<td>892</td>
<td>E.G. Kalnins, Willard Miller, Jr. and Sanchita Mukherjee</td>
<td>Models of $q$-algebra representations: the group of plane motions</td>
</tr>
<tr>
<td>893</td>
<td>T.R. Hoffend Jr. and R.K. Kaul</td>
<td>Relativistic theory of superpotentials for a nonhomogeneous, spatially isotropic medium</td>
</tr>
<tr>
<td>894</td>
<td>Reinhold von Schwerin</td>
<td>Two metal deposition on a microdisk electrode</td>
</tr>
<tr>
<td>895</td>
<td>Vladimir I. Oliker and Nina N. Ural'tseva</td>
<td>Evolution of nonparametric surfaces with speed depending on curvature, III. Some remarks on mean curvature and anisotropic flows</td>
</tr>
<tr>
<td>896</td>
<td>Wayne Barrett, Charles R. Johnson, Raphael Loewy and Tamir Shalom</td>
<td>Rank incrementation via diagonal perturbations</td>
</tr>
<tr>
<td>897</td>
<td>Mingxiang Chen, Xu-Yan Chen and Jack K. Hale</td>
<td>Structural stability for time-periodic one-dimensional parabolic equations</td>
</tr>
<tr>
<td>898</td>
<td>Hong-Ming Yin</td>
<td>Global solutions of Maxwell's equations in an electromagnetic field with the temperature-dependent electrical conductivity</td>
</tr>
<tr>
<td>899</td>
<td>Robert Grone, Russell Merris and William Watkins</td>
<td>Laplacian unimodular equivalence of graphs</td>
</tr>
<tr>
<td>900</td>
<td>Miroslav Fiedler</td>
<td>Structure-ranks of matrices</td>
</tr>
<tr>
<td>901</td>
<td>Miroslav Fiedler</td>
<td>An estimate for the nonstochastic eigenvalues of doubly stochastic matrices</td>
</tr>
<tr>
<td>902</td>
<td>Miroslav Fiedler</td>
<td>Remarks on eigenvalues of Hankel matrices</td>
</tr>
<tr>
<td>903</td>
<td>Charles R. Johnson, D.D. Olesky, Michael Tsatsomeros and P. van den Driessche</td>
<td>Spectra with positive elementary symmetric functions</td>
</tr>
<tr>
<td>904</td>
<td>Pierre-Alain Gremaud</td>
<td>Thermal contraction as a free boundary problem</td>
</tr>
<tr>
<td>905</td>
<td>K.L. Cooke, Janos Turi and Gregg Turner</td>
<td>Stabilization of hybrid systems in the presence of feedback delays</td>
</tr>
<tr>
<td>906</td>
<td>Robert P. Gilbert and Yongzhi Xu</td>
<td>A numerical transmutation approach for underwater sound propagation</td>
</tr>
<tr>
<td>907</td>
<td>LeRoy B. Beasley, Richard A. Brualdi and Bryan L. Shader</td>
<td>Combinatorial orthogonality</td>
</tr>
<tr>
<td>908</td>
<td>Richard A. Brualdi and Bryan L. Shader</td>
<td>Strong hall matrices</td>
</tr>
<tr>
<td>909</td>
<td>Håkan Wennerström and David M. Anderson</td>
<td>Difference versus Gaussian curvature energies; monolayer versus bilayer curvature energies applications to vesicle stability</td>
</tr>
<tr>
<td>910</td>
<td>Shmuels Friedland</td>
<td>Eigenvalues of almost skew symmetric matrices and tournament matrices</td>
</tr>
<tr>
<td>911</td>
<td>Avner Friedman, Bei Hu and J.L. Velazquez</td>
<td>A Free Boundary Problem Modeling Loop Dislocations in Crystals</td>
</tr>
<tr>
<td>912</td>
<td>Ezio Venturino</td>
<td>The Influence of Diseases on Lotka-Volterra Systems</td>
</tr>
<tr>
<td>913</td>
<td>Steve Kirkland and Bryan L. Shader</td>
<td>On Multivariate Tournament Matrices with Constant Team Size</td>
</tr>
<tr>
<td>914</td>
<td>Steve Kirkland and Bryan L. Shader</td>
<td>On Multivariate Tournament Matrices with Constant Team Size</td>
</tr>
</tbody>
</table>
Richard A. Brualdi and Jennifer J.Q. Massey, More on Structure-Ranks of Matrices

Douglas B. Meade, Qualitative Analysis of an Epidemic Model with Directed Dispersion

Kazuo Murota, Mixed Matrices Irreducibility and Decomposition

Richard A. Brualdi and Jennifer J.Q. Massey, Some Applications of Elementary Linear Algebra in Combinations

Carl D. Meyer, Sensitivity of Markov Chains

Hong-Ming Yin, Weak and Classical Solutions of Some Nonlinear Volterra Integrodifferential Equations

B. Leinikuhler and A. Ruehli, Exploiting Symmetry and Regularity in Waveform Relaxation Convergence Estimation

Xinfu Chen and Charles M. Elliott, Asymptotics for a Parabolic Double Obstacle Problem

Yongzhi Xu and Yi Yan, An Approximate Boundary Integral Method for Acoustic Scattering in Shallow Oceans

Yongzhi Xu and Yi Yan, Source Localization Processing in Perturbed Waveguides

Kenneth L. Cooke and Janos Turi, Stability, Instability in Delay Equations Modeling Human Respiration Maps Describing the Equilibrium of Nematic Phases Between Cylinders

F. Bethuel, H. Brezis, B.D. Coleman and F. Hélein, Bifurcation Analysis of Minimizing Harmonic

Frank W. Elliott, Jr., Signed Random Measures: Stochastic Order and Kolmogorov Consistency Conditions

D.A. Gregory, S.J. Kirkland and B.L. Shader, Pick’s Inequality and Tournaments

J.W. Demmel, N.J. Higham and R.S. Schreiber, Block LU Factorization

Victor A. Galaktionov and Juan L. Vazquez, Regional Blow-Up in a Semilinear Heat Equation with Convergence to a Hamilton-Jacobi Equation

Bryan L. Shader, Convertible, Nearly Decomposable and Nearly Reducible Matrices

Dianne P. O’Leary, Iterative Methods for Finding the Stationary Vector for Markov Chains

Nicholas J. Higham, Perturbation theory and backward error for $AX - XB = C$

Z. Strakos and A. Greenbaum, Open questions in the convergence analysis of the Lanczos process for the real symmetric eigenvalue problem

Zhaojun Bai, Error analysis of the Lanczos algorithm for the nonsymmetric eigenvalue problem

Pierre-Alain Gremaud, On an elliptic-parabolic problem related to phase transitions in shape memory alloys

Bojan Mohar and Neil Robertson, Disjoint essential circuits in toroidal maps

Bojan Mohar, Convex representations of maps on the torus and other flat surfaces

Bojan Mohar and Svatopluk Poljak Eigenvalues in combinatorial optimization

Richard A. Brualdi, Keith L. Chavey and Bryan L. Shader, Conditional sign-solvability

Roger Fosdick and Ying Zhang, The torsion problem for a nonconvex stored energy function

René Ferland and Gaston Giroux, An unbounded mean-field intensity model: Propagation of the convergence of the empirical laws and compactness of the fluctuations

Wei-Ming Ni and Izumi Takagi, Spike-layers in semilinear elliptic singular Perturbation Problems

Henk A. Van der Vorst and Gerard G.L. Sleijpen, The effect of incomplete decomposition preconditioning on the convergence of conjugate gradients

S.P. Hastings and L.A. Peletier, On the decay of turbulent bursts

Apostolos Hadjidimos and Robert J. Plemmons, Analysis of p-cyclic iterations for Markov chains

ÅBjörck, H. Park and L. Eldén, Accurate downdating of least squares solutions

E.G. Kalsins, William Miller, Jr. and G.C. Williams, Recent advances in the use of separation of variables methods in general relativity

G.W. Stewart, On the perturbation of LU, Cholesky and QR factorizations

G.W. Stewart, Gaussian elimination, perturbation theory and Markov chains

G.W. Stewart, On a new way of solving the linear equations that arise in the method of least squares

G.W. Stewart, On the early history of the singular value decomposition

G.W. Stewart, On the perturbation of Markov chains with nearly transient states

Umberto Mosco, Composite media and asymptotic dirichlet forms

Walter F. Mascarenhas, The structure of the eigenvectors of sparse matrices

Walter F. Mascarenhas, A note on Jacobi being more accurate than QR

Raymond H. Chan, James G. Nagy and Robert J. Plemmons, FFT-based preconditioners for Toeplitz-Block least squares problems

Zhaojun Bai, The CSD, GSVD, their applications and computations

D.A. Gregory, S.J. Kirkland and N.J. Pullman, A bound on the exponent of a primitive matrix using Boolean rank

Richard A. Brualdi, Shmuel Friedland and Alex Pothen, Sparse bases, elementary vectors and nonzero minors of compound matrices

J.W. Demmel, Open problems in numerical linear algebra

James W. Demmel and William Gragg, On computing accurate singular values and eigenvalues of acyclic matrices

James W. Demmel, The inherent inaccuracy of implicit tridiagonal QR

J.J.L. Velázquez, Estimates on the $(N - 1)$-dimensional Hausdorff measure of the blow-up set for a semilinear heat equation