

DISCRETIZATION OF MAXWELL'S EQUATIONS

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In electrodynamics we encounter the fields \mathbf{E} , \mathbf{B} , \mathbf{H} , \mathbf{D} , and when looking at the differential formulation of Maxwell's equations, one might feel tempted to treat them as “vectorfields with three components”. However, keeping in mind the integral form of Ampere's law and Faraday's law, we immediately see that the fields are abstractions for assigning real numbers to oriented submanifolds of the affine space \mathbb{R}^3 . In other words, the classical electromagnetic fields turn out to be generalized differential forms on \mathbb{R}^3 .

Surprisingly, this insight proves useful, when one aims to approximate electromagnetic fields. It leads to the conclusion that co-chains on cellular complexes are the perfect tool to capture most topological aspects of the laws of electromagnetism in a discrete setting. Yet, other aspects cannot be taken into account, especially the constitutive equations linking forms of different degree: the issue of how to introduce so-called *discrete Hodge operators* looms large.

Various solutions to the problem of suitable discrete constitutive equations have been suggested:

- the use of dual grids leading to a representation of discrete Hodge operators by means of a diagonal matrix.
- mimetic finite differences, which boil down to a lumping procedure.
- the passage from co-chains to genuine *discrete differential forms* by a judicious interpolation procedure.

The third option amounts to constructing conforming finite elements (FEM) for electromagnetic fields. They may look exotic, but neatly fit the framework numerical analysis provides for treating finite element schemes: as in the case of standard FEM we can construct higher order methods and investigate convergence.

References

- [1] A. BOSSAVIT, *Mixed finite elements and the complex of Whitney forms*, in The Mathematics of Finite Elements and Applications VI, J. Whiteman, ed., Academic Press, London, 1988, pp. 137–144.
- [2] —, *Generalized finite differences in computational electromagnetics*, in Geometric Methods for Computational Electromagnetics, F. Teixeira, ed., vol. 32 of PIER, EMW Publishing, Cambridge, MA, 2001, pp. 45–64.
- [3] R. HIPTMAIR, *Discrete Hodge operators: An algebraic perspective*, in Geometric Methods for Computational Electromagnetics, F. Teixeira, ed., vol. 32 of PIER, EMW Publishing, Cambridge, MA, 2001, pp. 247–269.
- [4] —, *Higher order Whitney forms*, in Geometric Methods for Computational Electromagnetics, F. Teixeira, ed., vol. 32 of PIER, EMW Publishing, Cambridge, MA, 2001, pp. 271–299.
- [5] —, *Finite elements in computational electromagnetism*, Acta Numerica, (2002), pp. 237–339.
- [6] J. HYMAN AND M. SHASHKOV, *Mimetic finite difference methods for Maxwell’s equations and the equations of magnetic diffusion*, in Geometric Methods for Computational Electromagnetics, F. Teixeira, ed., vol. 32 of PIER, EMW Publishing, Cambridge, MA, 2001, pp. 89–121.
- [7] F. TEIXEIRA, *Geometric aspects of the simplicial discretization of Maxwell’s equations*, in Geometric Methods for Computational Electromagnetics, F. Teixeira, ed., vol. 32 of PIER, EMW Publishing, Cambridge, MA, 2001, pp. 171–188.
- [8] F. TEIXEIRA AND W. CHEW, *Lattice electromagnetic theory from a topological viewpoint*, J. Math. Phys., 40 (1999), pp. 169–187.
- [9] E. TONTI, *Finite formulation of the electromagnetic field*, in Geometric Methods for Computational Electromagnetics, F. Teixeira, ed., vol. 32 of PIER, EMW Publishing, Cambridge, MA, 2001, pp. 1–44.