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Architectures and Algorithms for Distributed Generation Control of Inertia-Less AC Microgrids

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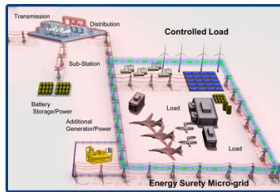
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Control at Large Scales: Energy Markets and Responsive Grids
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Outline

- 1 Introduction
- 2 Preliminaries
- 3 Generation Control Architecture
- 4 Distributed Implementation
- 5 Simulation Results
- 6 Concluding Remarks

Overview



Potential applications:

- Forward operating bases
- Disaster-relief installations
- Ultra reliable supply for data centers
- *All-electric* ship and *more-electric* aircraft
- Electrification of rural areas in developing countries

Focus of the Talk

The problem of generation control in islanded inertia-less AC microgrids

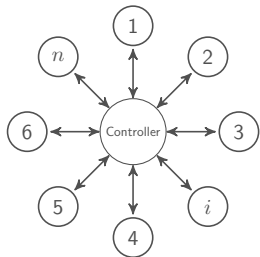
Architectural Solutions

Centralized:

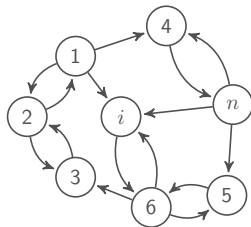
- ▶ Requires communication between a central processor and the various generation resources (and possibly loads)
- ▶ Requires up-to-date knowledge of generation resources availability
- ▶ Subject to failures at the decision maker (single-point-of-failure)

Distributed:

- ▶ Inherent ability to handle incomplete global knowledge
- ▶ Potential resiliency to faults and/or unpredictable behavior



Centralized



Distributed

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Modeling Assumptions

- A1. Short lossless lines
- A2. Balanced three-phase sinusoidal regime
- A3. Generators and loads interfaced through voltage source inverters
- A4. Inverter frequency and voltage magnitude droop-based controls
- A5. Voltage and current inner controller much faster than droop controls
- A6. Inverter reactive power capability sufficient to support voltage control
- A7. Voltage droop control much faster than frequency droop control
- A8. All quantities in p.u.
- A9. Base voltages = voltage control reference values

Assumptions A1, A5 – A9 \implies bus voltages approximately equal to 1 p.u.

Generator Buses

- For each generator bus $i \in \mathcal{V}_p^{(g)} := \{1, 2, \dots, m\}$:

$$D_i \frac{d\theta_i(t)}{dt} = u_i(t) - \sum_{j \in \mathcal{N}_p(i)} B_{ij} \sin(\theta_i(t) - \theta_j(t)),$$
$$\underline{u}_i \leq u_i(t) \leq \bar{u}_i,$$

where $\mathcal{N}_p(i)$ is the set of buses to which bus i is electrically connected

$\theta_i(t)$	Phase angle in reference frequency coordinate frame
$u_i(t)$	Active power set point
$\underline{u}_i, \bar{u}_i$	Set point lower and upper limits
D_i	Frequency control droop coefficient
B_{ij}	Absolute value of susceptance of line connecting nodes i and j

Load Buses

- For each load bus $i \in \mathcal{V}_p^{(\ell)} := \{m + 1, m + 2, \dots, n\}$:

$$D_i \frac{d\theta_i(t)}{dt} = -(\ell_i^0 + \Delta\ell_i(t)) - \sum_{j \in \mathcal{N}_p(i)} B_{ij} \sin(\theta_i(t) - \theta_j(t)),$$

where $\mathcal{N}_p(i)$ is the set of buses to which bus i is electrically connected

$\theta_i(t)$	Phase angle in reference frequency coordinate frame
ℓ_i^0	Nominal active power demand
$\Delta\ell_i(t)$	Active power demand perturbation
D_i	Frequency control droop coefficient
B_{ij}	Absolute value of susceptance of line connecting nodes i and j

Generation Control Objectives

O1. For ℓ_i^0 , $i \in \mathcal{V}_p^{(\ell)}$, find constant generator set points, u_i^* , $i \in \mathcal{V}_p^{(g)}$, so that

P1. $\sum_{i \in \mathcal{V}_p^{(g)}} u_i^* = \sum_{i \in \mathcal{V}_p^{(\ell)}} \ell_i^0$, $\underline{u}_i \leq u_i^* \leq \bar{u}_i$, $i \in \mathcal{V}_p^{(g)}$,

and there is a corresponding equilibrium point, θ_i^* , $i \in \mathcal{V}_p^{(g)} \cup \mathcal{V}_p^{(\ell)}$, satisfying

P2. $|\theta_i^* - \theta_j^*| \leq \phi$ for all i, j electrically connected, and $\phi \in [0, \pi/2)$

O2. For sufficiently small changes in $\Delta \ell_i(t)$, $i \in \mathcal{V}_p^{(\ell)}$, regulate the value of $u_i(t)$, $i \in \mathcal{V}_p^{(g)}$ around u_i^* , $i \in \mathcal{V}_p^{(g)}$ so that

P3. $\frac{d\theta_i(t)}{dt} \rightarrow 0$ as $t \rightarrow \infty$

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Overview of Frequency Regulation Architecture

- For given ℓ_i^0 's, u_i^* 's are computed to meet the properties in Objective O1
- After a load perturbation occurs, set points are iteratively adjusted around the u_i^* 's via closed-loop control
- These adjustments serve to eliminate the frequency error that results from the load perturbation
- In general, load perturbations and subsequent set point adjustments move the system away from the stable equilibrium point corresponding to the u_i^* 's
- The u_i^* 's are recomputed based upon an estimate for the amount by which the system has deviated from the aforementioned stable equilibrium point

Choosing Set Points

- For ℓ_i^0 , $i \in \mathcal{V}_p^{(\ell)}$, find constant generator set points, u_i^* , $i \in \mathcal{V}_p^{(g)}$, so that

$$\text{P1. } \sum_{i \in \mathcal{V}_p^{(g)}} u_i^* = \sum_{i \in \mathcal{V}_p^{(\ell)}} \ell_i^0, \quad \underline{u}_i \leq u_i^* \leq \bar{u}_i, \quad i \in \mathcal{V}_p^{(g)}$$

and there is a corresponding equilibrium point, θ_i^* , $i \in \mathcal{V}_p^{(g)} \cup \mathcal{V}_p^{(\ell)}$, satisfying

$$\text{P2. } |\theta_i^* - \theta_j^*| \leq \phi, \text{ for all } i, j, \text{ electrically connected, and } \phi \in [0, \pi/2)$$

The u_i^* 's can be obtained by solving a feasibility problem involving

- U1. Node power balance equations
- U2. Set point lower and upper limits

Feasibility Problem Formulation

- Let $\mathcal{G}_p = (\mathcal{V}_p, \mathcal{E}_p)$ be an undirected graph, where
 - $\mathcal{V}_p = \mathcal{V}_p^{(g)} \cup \mathcal{V}_p^{(\ell)}$,
 - $\mathcal{E}_p \subseteq \mathcal{V}_p \times \mathcal{V}_p$, with $\{i, j\} \in \mathcal{E}_p$ if nodes i and j are electrically connected
- Then, the u_i^* 's can be obtained as a solution of

$$\begin{aligned} & \text{find } u^*, \theta^* \\ & \text{subject to } \begin{bmatrix} u^* \\ -\ell^0 \end{bmatrix} = MB \mathbf{sin}(M^T \theta^*) \\ & \|M^T \theta^*\|_\infty \leq \phi \\ & \underline{u} \leq u^* \leq \bar{u}, \end{aligned}$$

 $u^* \in \mathbb{R}^m$

Vector of active power set points

$\theta^* \in \mathbb{R}^n$

Vector of phase angles

$\ell^0 \in \mathbb{R}^{n-m}$

Vector of nominal active power demands

$\underline{u}, \bar{u} \in \mathbb{R}^m$

Vector of set point lower and upper limits

$M \in \mathbb{R}^{n \times |\mathcal{E}_p|}$

Graph Incidence matrix for arbitrary edge orientation

$B \in \mathbb{R}^{|\mathcal{E}_p| \times |\mathcal{E}_p|}$

Diagonal matrix of line susceptance absolute values

$\mathbf{sin}(M^T \theta^*) \in \mathbb{R}^{|\mathcal{E}_p|}$

Vector of angle across line sine's

Flow Space Formulation

- The feasibility problem

$$\begin{aligned} \text{F0:} \quad & \text{find } u^*, \theta^* \\ & \text{subject to } \begin{bmatrix} u^* \\ -\ell^0 \end{bmatrix} = MB \sin(M^T \theta^*) \\ & \|M^T \theta^*\|_\infty \leq \phi \\ & \underline{u} \leq u^* \leq \bar{u}, \end{aligned}$$

is equivalent to

$$\begin{aligned} \text{F1:} \quad & \text{find } u^*, f^* \\ & \text{subject to } \begin{bmatrix} u^* \\ -\ell^0 \end{bmatrix} = M f^* \\ & \text{Narcsin}(B^{-1} f^*) = \mathbf{0} \\ & -\sin(\phi)b \leq f^* \leq \sin(\phi)b \\ & \underline{u} \leq u^* \leq \bar{u} \end{aligned}$$

$f^* \in \mathbb{R}^{ \mathcal{E}_p }$	Vector of line flows
$b \in \mathbb{R}^{ \mathcal{E}_p }$	Vector of line susceptance absolute values
$N \in \mathbb{R}^{(\mathcal{E}_p - n + 1) \times \mathcal{E}_p }$	Graph fundamental loop matrix
$\arcsin(B^{-1} f^*) \in \mathbb{R}^{ \mathcal{E}_p }$	Vector of normalized line flows arcsin's

Convexifying the Problem [Zholbaryssov, D-G, '16]

The constraint $N \arcsin(B^{-1}f^*) = \mathbf{0}$ makes feasibility problem F1 non convex

- **Trees:** Nothing to do as $N = 0$
- **Networks with non-overlapping loops:**
 - ▶ Remove $N \arcsin(B^{-1}f^*) = 0$, and
 - ▶ Replace $-\sin(\phi)b \leq f^* \leq \sin(\phi)b$ by

$$\underline{f} \leq f^* \leq \bar{f},$$

with $\bar{f} = -\underline{f} = \sin(\phi) (b - |N^T|\beta)$, where $\beta = [\beta_1, \dots, \beta_{|\mathcal{E}_p|-n+1}]$, and

$$\beta_i = \frac{1}{2} \left(\bar{b}_i - \sin \left(\frac{\phi}{|\mathcal{C}_i| - 1} \right) b_i \right)$$

\bar{b}_i	Maximum of line susceptance absolute values along loop i
b_i	Minimum of of line susceptance absolute values along loop i
$ \mathcal{C}_i $	Number of lines in loop i

Set Point Computation

- Feasibility Problem F0:

$$\begin{aligned} &\text{find } u^*, \theta^* \\ &\text{subject to } \begin{bmatrix} u^* \\ -\ell^0 \end{bmatrix} = MB \sin(M^T \theta^*) \\ &\quad \|M^T \theta^*\|_\infty \leq \phi \\ &\quad \underline{u} \leq u^* \leq \bar{u} \end{aligned}$$

- Feasibility Problem F2:

$$\begin{aligned} &\text{find } u^*, f^* \\ &\text{subject to } \begin{bmatrix} u^* \\ -\ell^0 \end{bmatrix} = M f^* \\ &\quad \underline{f} \leq f^* \leq \bar{f} \\ &\quad \underline{u} \leq u^* \leq \bar{u} \end{aligned}$$

Lemma

For every solution (u, f) of F2, there exists a θ so that (u, θ) is a solution of F0

- F2 is a special case of the class of problems studied in commodity networks
- F2 can be used to find set points that satisfy desired properties P1 and P2

Average Frequency Error

- Recall microgrid dynamic model:

$$D_i \frac{d\theta_i(t)}{dt} = u_i(t) - \sum_{j \in \mathcal{N}_p(i)} B_{ij} \sin(\theta_i(t) - \theta_j(t)), \quad i \in \mathcal{V}_p^{(g)}$$

$$D_i \frac{d\theta_i(t)}{dt} = -(\ell_i^0 + \Delta\ell_i(t)) - \sum_{j \in \mathcal{N}_p(i)} B_{ij} \sin(\theta_i(t) - \theta_j(t)), \quad i \in \mathcal{V}_p^{(\ell)}$$

- Define the average frequency error at $t \geq 0$ as follows

$$\Delta\bar{\omega}(t) := \frac{\sum_{i=1}^n D_i \frac{d\theta_i(t)}{dt}}{\sum_{i=1}^n D_i}$$

- Then, by adding the microgrid dynamics equations

$$\Delta\bar{\omega}(t) = \frac{1}{\sum_{i=1}^n D_i} \left(\sum_{i \in \mathcal{V}_p^{(g)}} u_i(t) - \sum_{i \in \mathcal{V}_p^{(\ell)}} (\ell_i^0 + \Delta\ell_i(t)) \right)$$

Control Scheme [Cady, Zholbaryssov, D-G, Hadjicostis, '16]

- The intervals of control scheme execution are referred to as *rounds*
 - ▶ Rounds are indexed by $r = 0, 1, 2, \dots$
 - ▶ The duration of each round is T_0 units of time
 - ▶ $t_r := rT_0$ denotes the time at the beginning of round r
- Assume a large number of rounds elapsed between load perturbations, i.e.,
 - ▶ If the power demanded by load $i \in \mathcal{V}_p^{(\ell)}$ is perturbed by $\Delta\ell_i$ at $t = t_0$, then $\ell_i(t) = \ell_i^0 + \Delta\ell_i$ for $t_0 < t < r_0T_0$ and r_0 sufficiently large
- Let $u_i[r] := u_i(t)$, $t_r \leq t < t_{r+1}$, then

$$\Delta\bar{\omega}[r] = \bar{D} \left(\sum_{i \in \mathcal{V}_p^{(g)}} u_i[r] - \sum_{i \in \mathcal{V}_p^{(\ell)}} (\ell_i^0 + \Delta\ell_i) \right),$$

where $\bar{D} := \frac{1}{\sum_{i=1}^n D_i}$

Control Objective

Iteratively adjust set-points so as to drive the average frequency error to zero

Control Scheme [Cady, Zholbarysov, D-G, Hadjicostis, '16

- For a given u_i^* , generator i adjusts its set point as follows:

$$\begin{aligned}e_i[r+1] &= e_i[r] + \kappa_i \Delta \bar{\omega}[r], \\ u_i[r] &= u_i^* + \alpha_i e_i[r],\end{aligned}$$

where $e_i[0] = 0$, and α_i and κ_i are appropriately chosen gains

- Let $e[r] = [e_1[r], e_2[r], \dots, e_m[r]]^T$, then:

$$\begin{bmatrix} \Delta \bar{\omega}[r+1] \\ e[r+1] \end{bmatrix} = \underbrace{\begin{bmatrix} \beta & \bar{D}\alpha^T \\ \kappa & I_m \end{bmatrix}}_{:=\Phi} \begin{bmatrix} \Delta \bar{\omega}[r] \\ e[r] \end{bmatrix} + \begin{bmatrix} -\bar{D} \\ 0_m \end{bmatrix} \sum_{i \in \mathcal{V}_p^{(\ell)}} \Delta \ell_i,$$

where $\beta := \bar{D} \sum_{i \in \mathcal{V}_p^{(g)}} \alpha_i \kappa_i$, $\alpha := [\alpha_1, \dots, \alpha_m]^T$, and $\kappa := [\kappa_1, \dots, \kappa_m]^T$

Gain Choice

Proposition

If α_i and κ_i for $i \in \mathcal{V}_p^{(g)}$ are chosen such that

$$-2 < \bar{D} \sum_{i \in \mathcal{V}_p^{(g)}} \alpha_i \kappa_i < 0,$$

then

- The system is marginally stable and the average frequency error asymptotically approaches zero, i.e., $\rho(\Phi) \leq 1$ and $\Delta\bar{\omega}[r] \rightarrow 0$ as $r \rightarrow \infty$
- The value of the average frequency error at any round r is given by

$$\Delta\bar{\omega}[r] = (1 + \beta)^{r-1}(\beta - 1)\bar{D} \left(\sum_{i \in \mathcal{V}_p^{(\ell)}} \Delta\ell_i \right)$$

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Control Dependence on Global Information

- Recall the set-point adjustment scheme::

$$\begin{aligned}e_i[r + 1] &= e_i[r] + \kappa_i \Delta \bar{\omega}[r], \\ u_i[r] &= u_i^* + \alpha_i e_i[r],\end{aligned}$$

where $e_i[0] = 0$, and α_i and κ_i chosen so that $-2 < \bar{D} \sum_{i \in \mathcal{V}_p^{(g)}} \alpha_i \kappa_i < 0$

- In order to implement and execute this control, each generator requires some global information:
 - Q1. Set point u_i^*
 - Q2. Gains α_i and κ_i
 - Q3. Average frequency error $\Delta \bar{\omega}[r]$

Objective

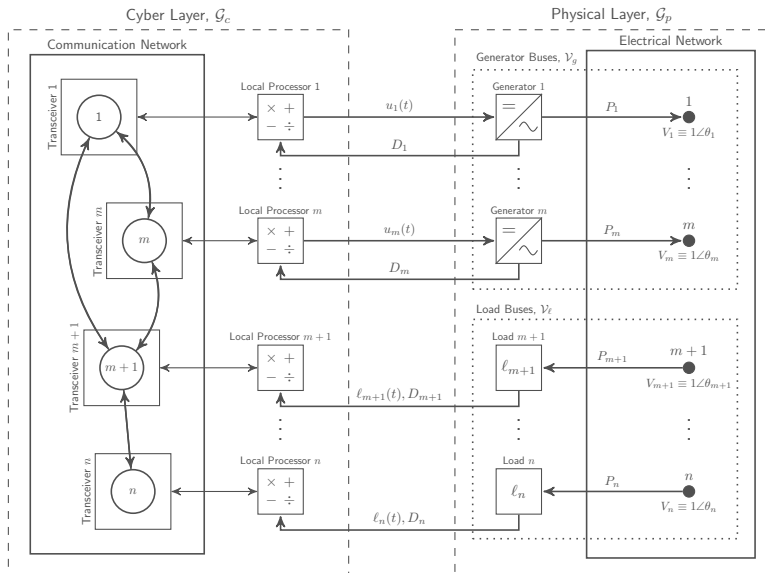
Design distributed algorithms that enables the computation of Q1-Q3

Cyber Layer for Distributed Computation and Control

- Bus local processors can perform simple computations ($\times, \div, +, -$)
- Exchange of information between local processors is described by a **connected undirected graph** $\mathcal{G}_c = \{\mathcal{V}_c, \mathcal{E}_c\}$:
 - ▶ $\mathcal{V}_c := \{1, 2, \dots, n\}$ is the set of nodes
 - ▶ $\mathcal{E}_c \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, where $\{i, j\} \in \mathcal{E}_c$ if nodes i and j can exchange information
 - ▶ Each element in \mathcal{V}_c corresponds to a bus of the physical network, i.e., there is a one-to-one mapping between \mathcal{V}_c and \mathcal{V}_p
 - ▶ Each element in \mathcal{E}_c corresponds to a line of the physical network, i.e., there is a one-to-one mapping between \mathcal{E}_c and \mathcal{E}_p
- Nodes that exchange information with node i are said to be its neighbors and are represented by the set

$$\mathcal{N}_c(i) = \{i \in \mathcal{V} : \{i, j\} \in \mathcal{E}_c\}$$

Interaction Between Physical and Cyber Layers



Set Point Computation [Cady, Hadjicostis, D-G, '15]

- Recall feasibility problem F2:

$$\begin{aligned} & \text{find } u^*, f^* \\ & \text{subject to } \begin{bmatrix} u^* \\ -\ell^0 \end{bmatrix} = Mf^* \\ & \underline{f} \leq f^* \leq \bar{f} \\ & \underline{u} \leq u^* \leq \bar{u} \end{aligned}$$

- Distributed algorithm over graph \mathcal{G}_c for solving F2:
 - Each node $i \in \mathcal{V}_p$ maintains and updates estimates of flows with its neighbors:
 $f_{ij}^{(i)}[k]$, $j \in \mathcal{N}_p(i)$, $k = 0, 1, 2, \dots$
 - Each generator node $i \in \mathcal{V}_p^{(g)}$ maintains an estimate for its set point:
 $u_i[k]$, $k = 0, 1, 2, \dots$
 - At each iteration k , each node $i \in \mathcal{V}_p$ computes its **power flow balance**:

$$d_i[k] := \begin{cases} -\sum_{j \in \mathcal{N}_p(i)} f_{ij}^{(i)}[k] + u_i[k], & i \in \mathcal{V}_p^{(g)} \\ -\sum_{j \in \mathcal{N}_p(i)} f_{ij}^{(i)}[k] - \ell_i, & i \in \mathcal{V}_p^{(\ell)} \end{cases}$$

Algorithm Progress

Each node iteratively updates its estimates so as to drive its power flow balance to zero

Iteration Three-Step Process

- **[Step 1]** Estimates are adjusted to interim values using the flow balance:

$$\begin{aligned}\tilde{f}_{ij}^{(i)}[k+1] &= f_{ij}^{(i)}[k] + \frac{d_i[k]}{\mathcal{N}_p(i) + 1} \\ \hat{u}_i[k+1] &= u_i[k] - \frac{d_i[k]}{2(\mathcal{N}_p(i) + 1)}\end{aligned}$$

- **[Step 2]** Pairs of neighbors average interim estimates of flow between them:

$$\hat{f}_{ij}^{(i)}[k+1] = \frac{1}{2} \left(\tilde{f}_{ij}^{(i)}[k+1] - \tilde{f}_{ji}^{(j)}[k+1] \right)$$

- **[Step 3]** New estimate values are obtained by enforcing limits:

$$\begin{aligned}f_{ij}^{(i)}[k+1] &= \left[\hat{f}_{ij}^{(i)}[k+1] \right]_{\underline{f}_{ij}}^{\bar{f}_{ij}} \\ u_i[k+1] &= \left[\hat{u}_i[k+1] \right]_{\underline{u}_i}^{\bar{u}_i}\end{aligned}$$

Feasible Flow Algorithm Convergence

- Since $f_{ij}^{(i)}[k] = -f_{ji}^{(j)}[k] =: f_{ij}[k]$, Steps 1-3 can be collapsed:

$$f_{ij}[k+1] = \left[f_{ij}[k] + \frac{1}{2} \left(\frac{d_i[k]}{\mathcal{N}_p(i) + 1} - \frac{d_j[k]}{\mathcal{N}_p(j) + 1} \right) \right]_{\underline{f}_{ij}}^{\bar{f}_{ij}} \quad (1)$$

$$u_i[k+1] = \left[u_i[k] - \frac{d_i[k]}{2(\mathcal{N}_p(i) + 1)} \right]_{\underline{u}_i}^{\bar{u}_i} \quad (2)$$

Proposition

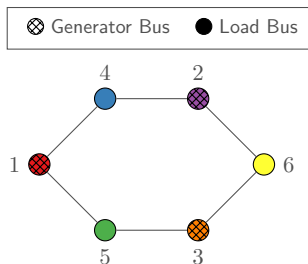
Suppose there exists a solution, (u^*, f^*) , to feasibility problem F2. Then, as $k \rightarrow \infty$, $u_i \rightarrow u_i^*$, $f_{ij}[k] \rightarrow f_{ij}^*$, $d_i[k] \rightarrow 0$

- Convergence proof relies on the observation that the iterations in (1) - (2) corresponds to those of a gradient descent algorithm for a quadratic program

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Six-Node Network Case Study



Generator bus parameters

i	\underline{u}_i	\bar{u}_i	D_i
1	0.1	1.15	0.225
2	0.15	2.65	0.679
3	0.05	1.68	0.95

Load bus parameter

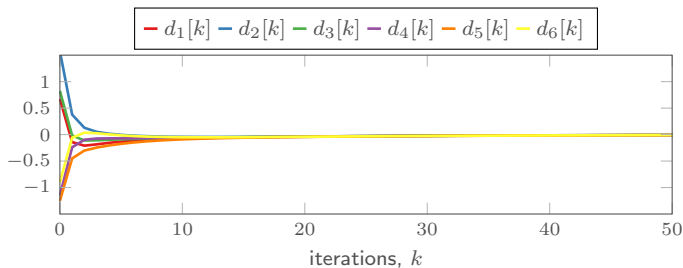
i	ℓ_i^0	D_i
4	1.15	0.0125
5	1.25	0.0679
6	0.9	0.0479

Line parameters

Line Index	{1, 4}	{1, 5}	{2, 4}	{2, 6}	{3, 5}	{3, 6}
Susceptance	-2.919	-6.685	-4.474	-4.375	-7.435	-6.274

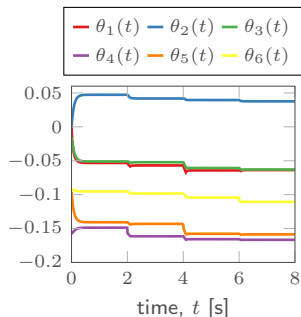
Initial Set-Point Computation

- Flow balances converge to within ± 0.001 in the first 50 iterations, yielding:
 - ▶ $u_1^* = 0.85$
 - ▶ $u_2^* = 1.5$ pu
 - ▶ $u_3^* = 0.95$ pu

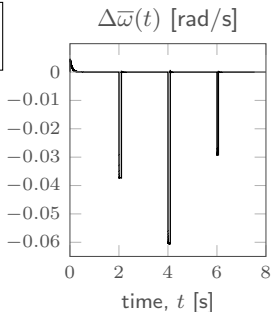


System Response to Load Perturbations

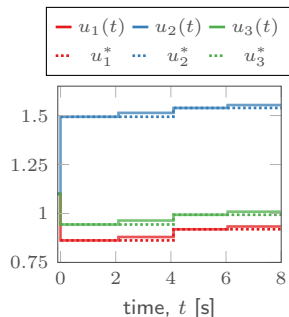
- l_4 , l_6 , and l_5 perturbed at $t = 2$ s, $t = 4$ s, and $t = 6$ s, respectively:
 - ▶ $\Delta l_4 = 0.0575$ pu
 - ▶ $\Delta l_5 = 0.09375$ pu
 - ▶ $\Delta l_6 = 0.045$ pu
- u_1^* , u_2^* , u_3^* recomputed at $t = 2$ s



(a) Bus voltage angles



(b) Average frequency error



(c) Set-points

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Summary

- Things discussed today:
 - ▶ Architecture for frequency regulation in islanded inertia-less AC microgrids
 - ▶ Computation of set points guaranteed to yield a stable equilibrium point
 - ▶ Distributed algorithm for set-point computation
- Things not discussed today [Cady, Zholbaryssov, D-G, Hadjicostis, '16]:
 - ▶ Distributed computation of controller gains
 - ▶ Distributed computation of average frequency error
 - ▶ Criteria and distributed protocol for triggering for set-point recomputation
- Future work:
 - ▶ Voltage control architecture
 - ▶ Extensions to lossy networks