FD AND TD SCATTERING FROM DNG SLABS

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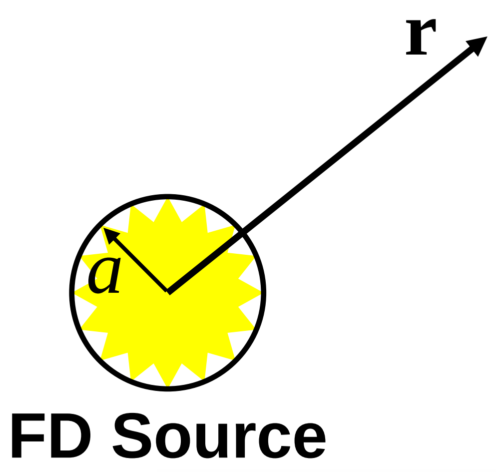
&
Thorkild B. Hansen

(Physical Review E, April 2006)

ENERGY IN TIME-HARMONIC FIELDS



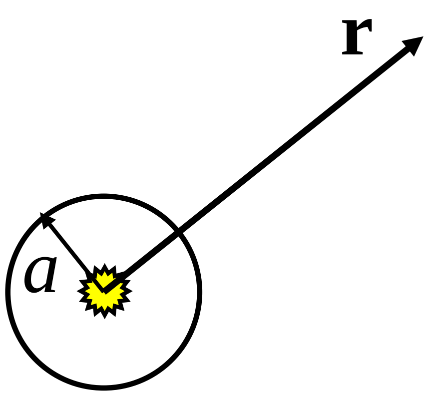
$$\mathbf{F}(\theta,\phi) = \lim_{r\to\infty} r e^{jkr}\mathbf{E}(\mathbf{r})$$

$$\mathcal{W} = \frac{1}{4}\int_{4\pi}\int_{a}^{\infty}\left[\epsilon_0|\mathbf{E}(\mathbf{r})|^2 + \mu_0|\mathbf{H}(\mathbf{r})|^2\right] r^2 d\Omega dr$$

$$= \frac{1}{2}\epsilon_0 \int_{a}^{\infty} |\mathbf{F}(\mathbf{r})|^2 d\Omega dr = \infty$$

!!

REACTIVE ENERGY IN SPHERICAL MULTIPOLE

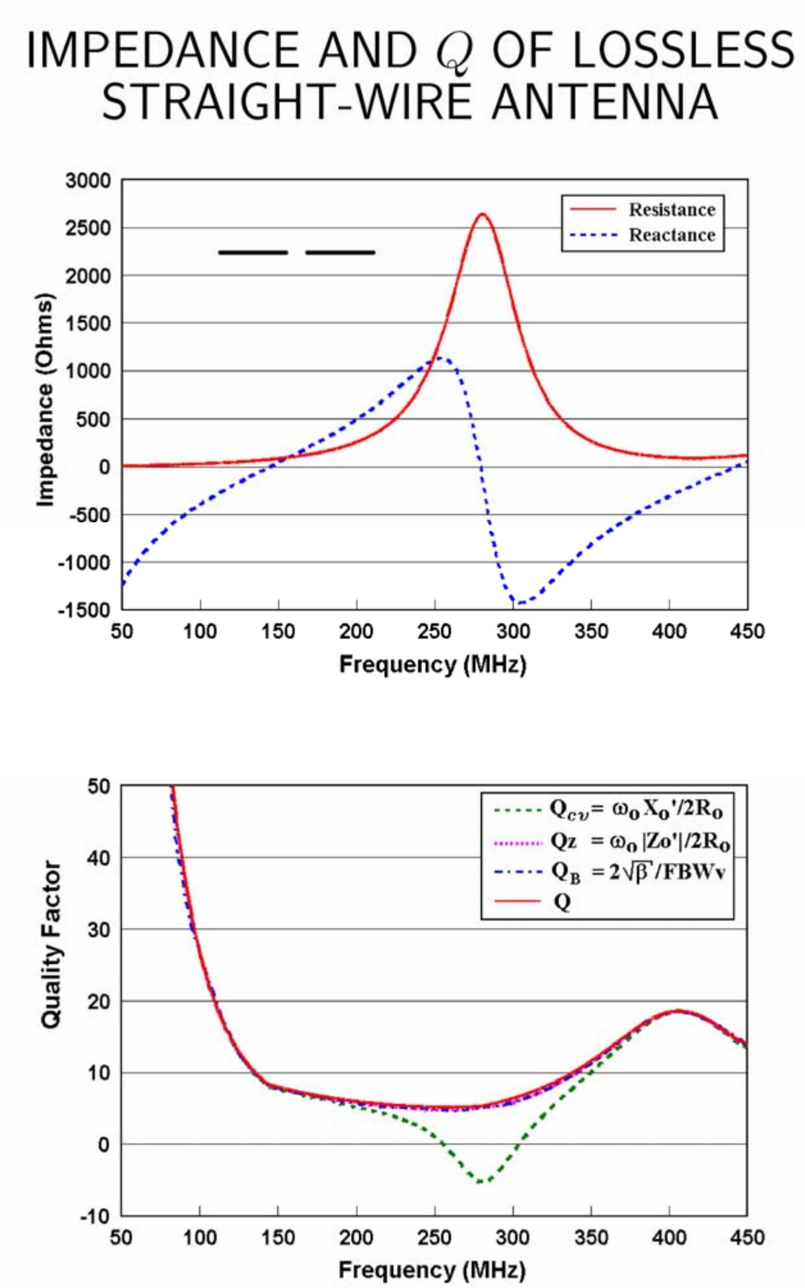


FD Source

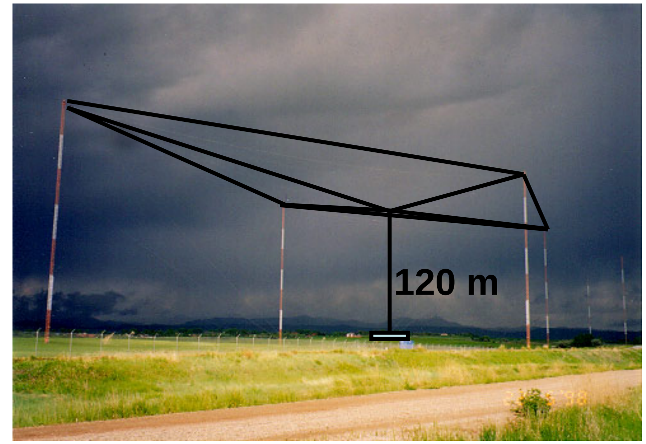
$$\mathbf{F}(\theta, \phi) = \lim_{r \to \infty} r e^{jkr} \mathbf{E}(\mathbf{r})$$

$$W = \frac{1}{4} \int_{4\pi} \int_{0}^{a} \left[\epsilon_0 |\mathbf{E}(\mathbf{r})|^2 + \mu_0 |\mathbf{H}(\mathbf{r})|^2 r^2 - 2\epsilon_0 |\mathbf{F}(\mathbf{r})|^2 \right] d\Omega dr = \infty$$

!!



More than you wanted to know about Q:
Yaghjian (AWPL, April 2006)
Yaghjian and Best (AP-S Trans., April 2005)



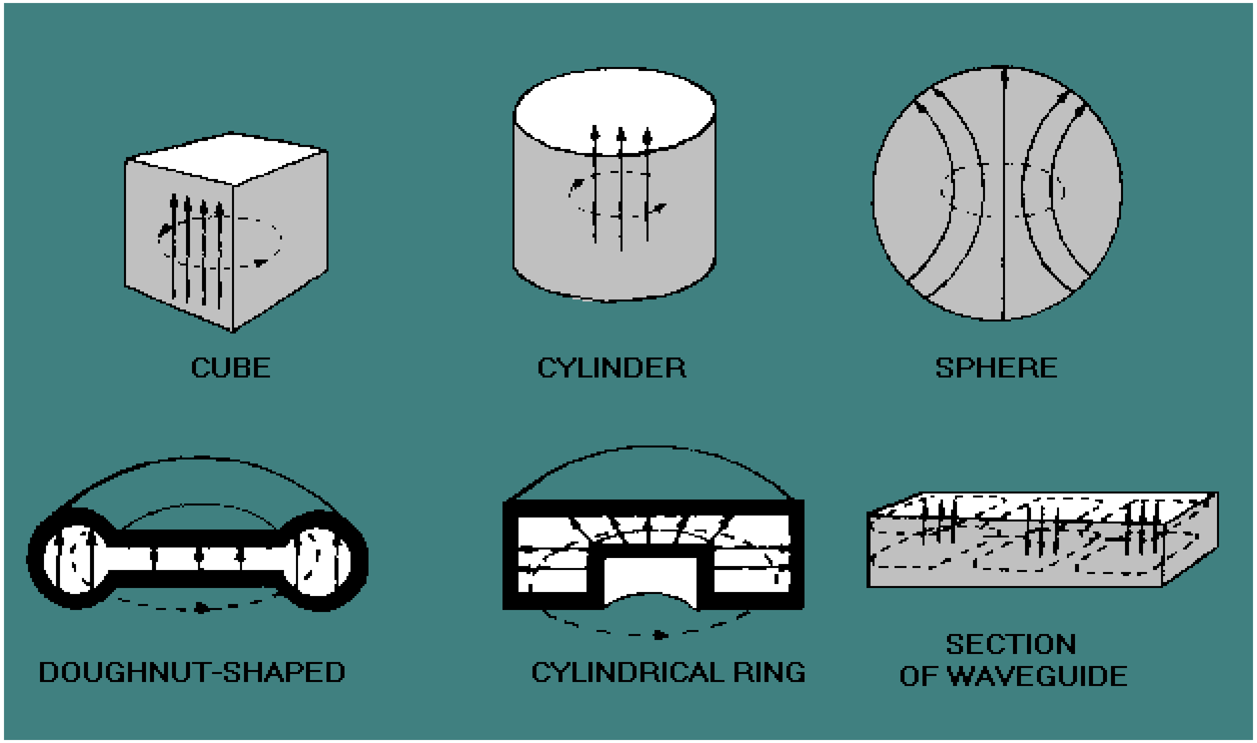
NIST 60 kHz Antenna
(Fort Collins, CO)

High Reactive Energy

⬇

High reactance
High quality factor (Q)
Low bandwidth
Low radiation resistance
Low power radiated
Low radiation efficiency

FINITE CW POWER CAN PRODUCE INFINITE REACTIVE POWER

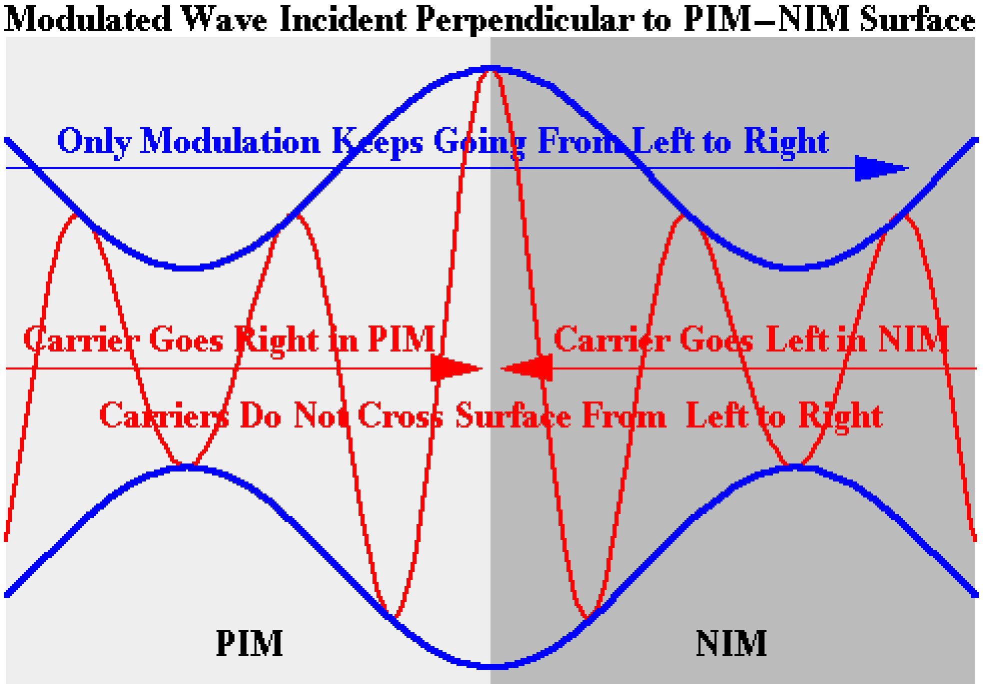


SOPRANO AND THE WINE GLASS

Relatively low power at a resonant frequency can shatter a wine glass.

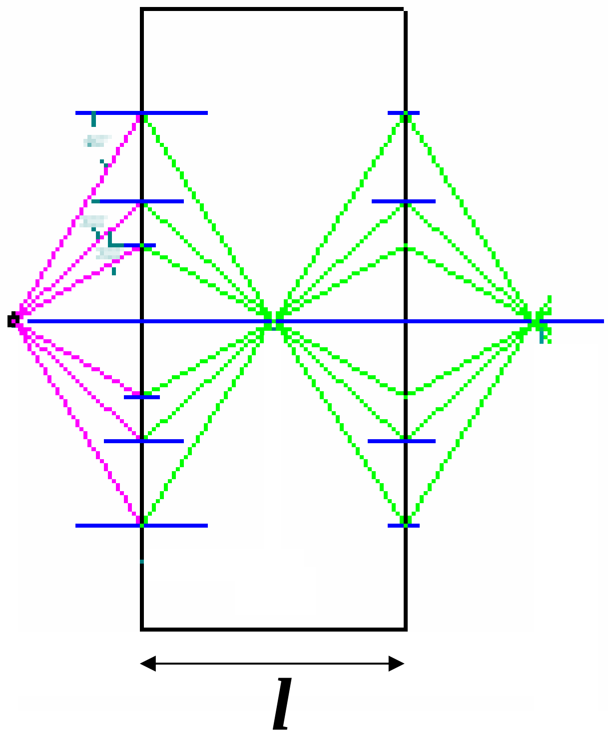
Anti-Parallel Phase Velocity and Energy Flow



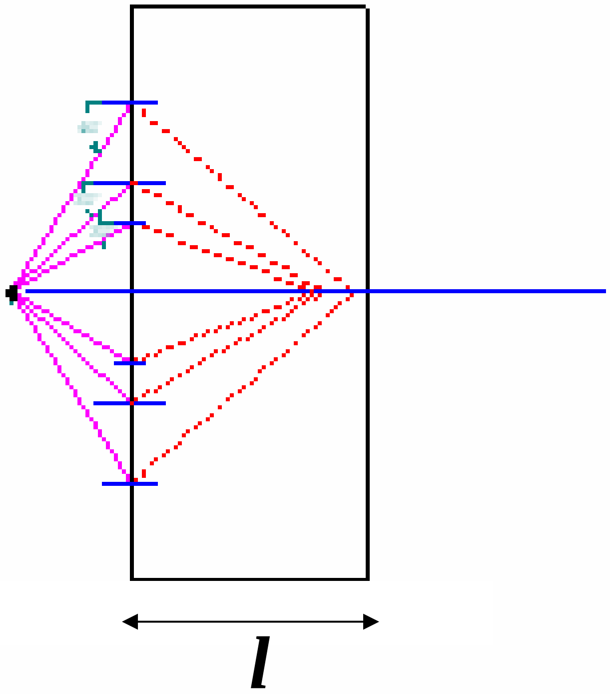
Animation courtesy of: University of Texas, Austin Center for Electromagnetic Materials and Devices

RAY TRACING (PROPAGATING WAVES)

$$\varepsilon / \varepsilon_0 = \mu / \mu_0 = -1$$

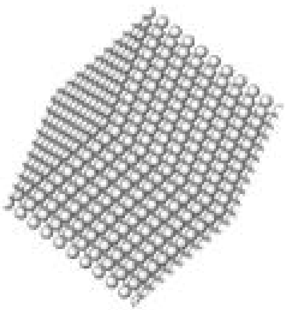


$$\varepsilon / \varepsilon_0 = \mu / \mu_0 = -1.5$$

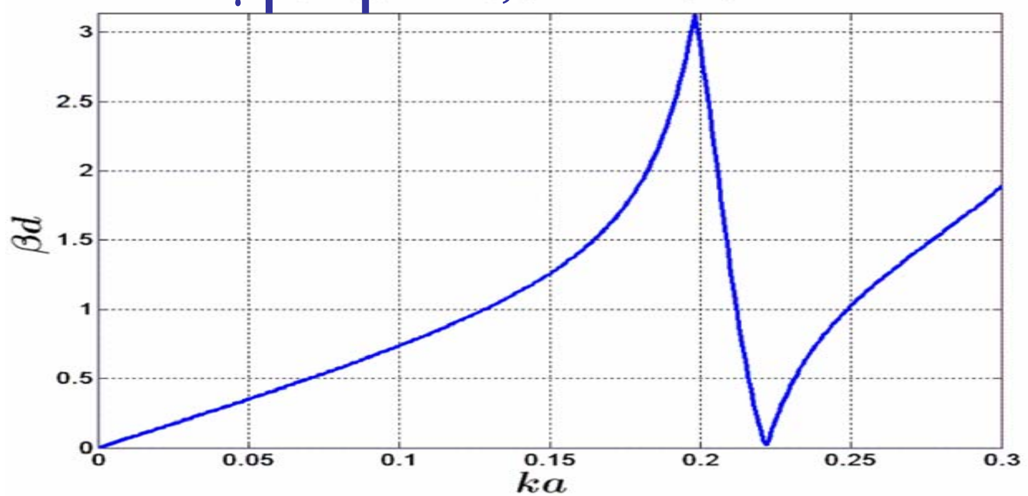


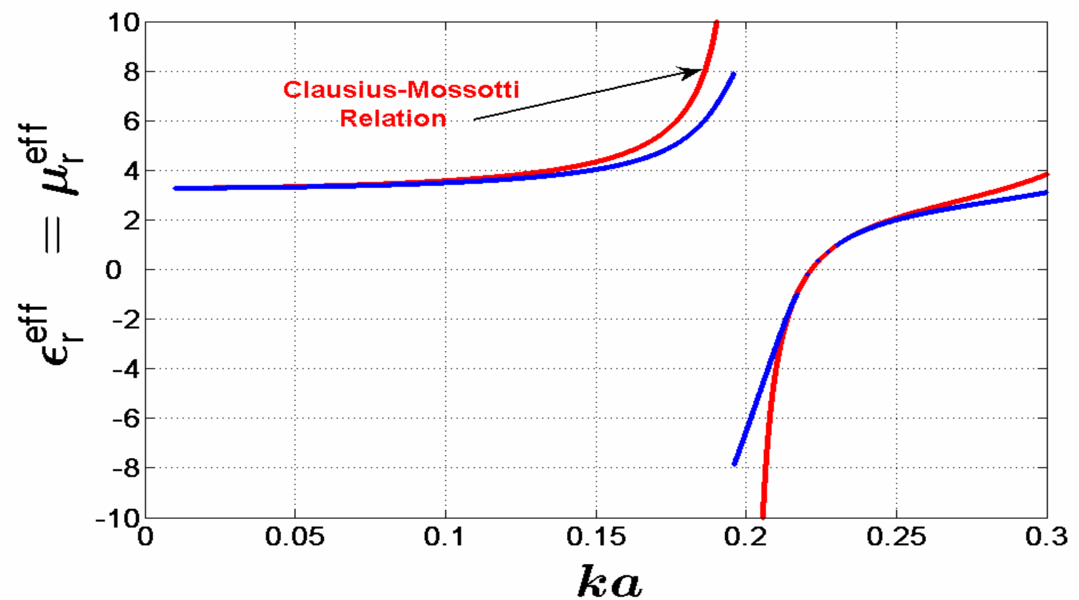
3-D ARRAY OF MAGNETODIELECTRIC SPHERES

(Shore and Yaghjian: URSI GA, Oct 2005; Inhouse Rpt. Sept 2006)

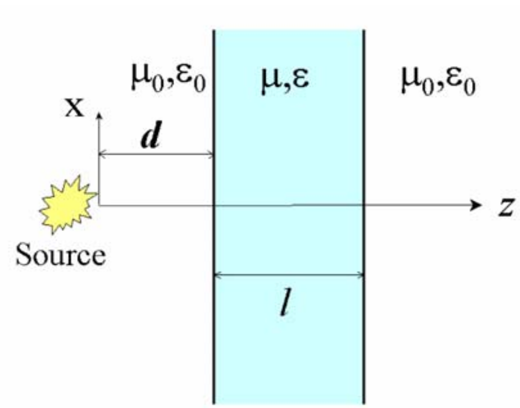


$\mu_r = \varepsilon_r = 20$, a / d = .4924





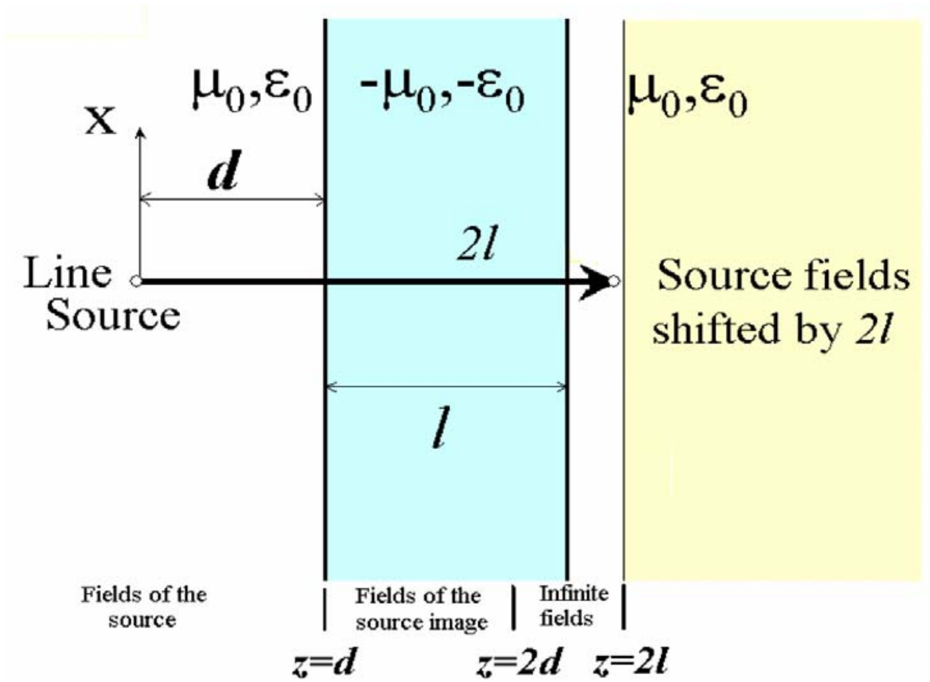
LOSSLESS SLAB



$$\mathcal{T}_{TE}(k_x) = \frac{4e^{-i\gamma_0 l}}{\left(2 + \frac{\epsilon\gamma_0}{\epsilon_0\gamma} + \frac{\epsilon_0\gamma}{\epsilon\gamma_0}\right) e^{-i\gamma l} + \left(2 - \frac{\epsilon\gamma_0}{\epsilon_0\gamma} - \frac{\epsilon_0\gamma}{\epsilon\gamma_0}\right) e^{i\gamma l}}$$

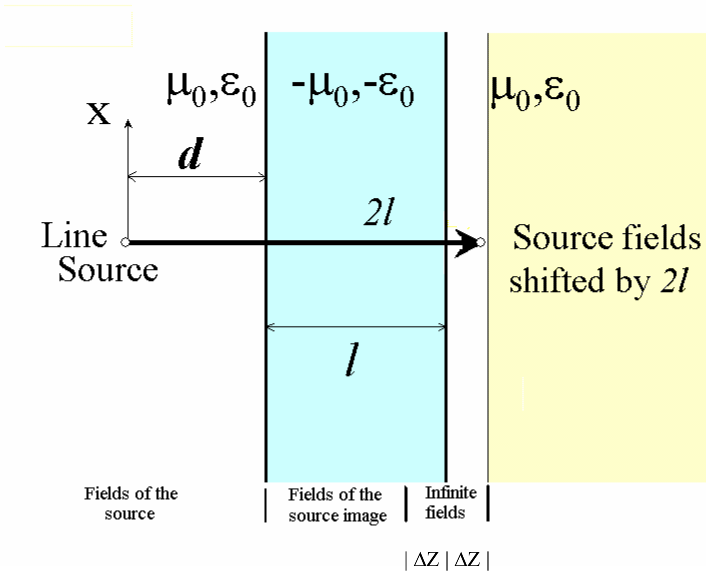
$$E_x^{\mathbf{tr}}(x, z) = \int_{-\infty}^{+\infty} T_0(k_x)\mathcal{T}_{TE}(k_x)e^{i(\gamma_0 z + k_x x)}dk_x$$

$$\text{If } \ (\mu\,, \epsilon) = (-\mu_0\,, -\epsilon_0) \ \text{ then } \ \mathcal{T}_{TE}(k_x) = e^{-i\gamma_0 2l}$$



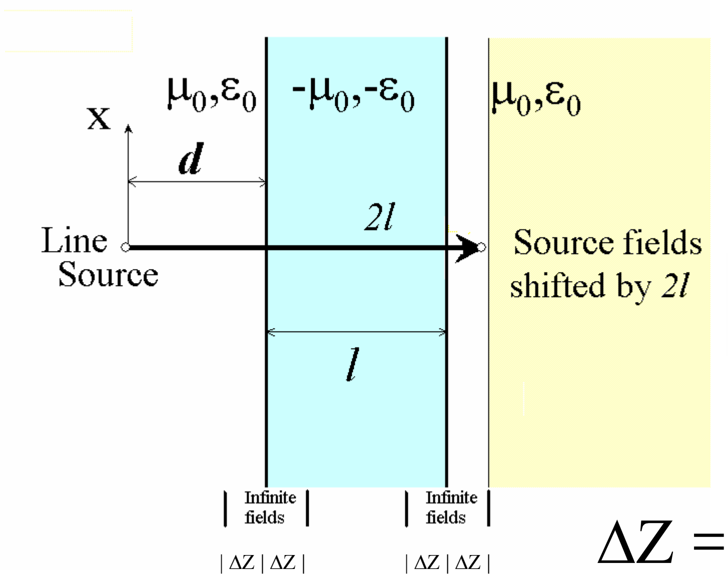
Noise prevents super resolution of sources further than about λ/2 from the slab. Slab thickness is limited by loss. Finite bandwidth, finite slab height, and inhomogeneities reduce resolution.

LOSSY SLAB



$$E_x^{\mathbf{tr}}(x,z) = \int_{-\infty}^{+\infty} T_0(k_x)\mathcal{T}_{TE}(k_x)e^{i(\gamma_0 z + k_x x)}dk_x \qquad z > d + l$$

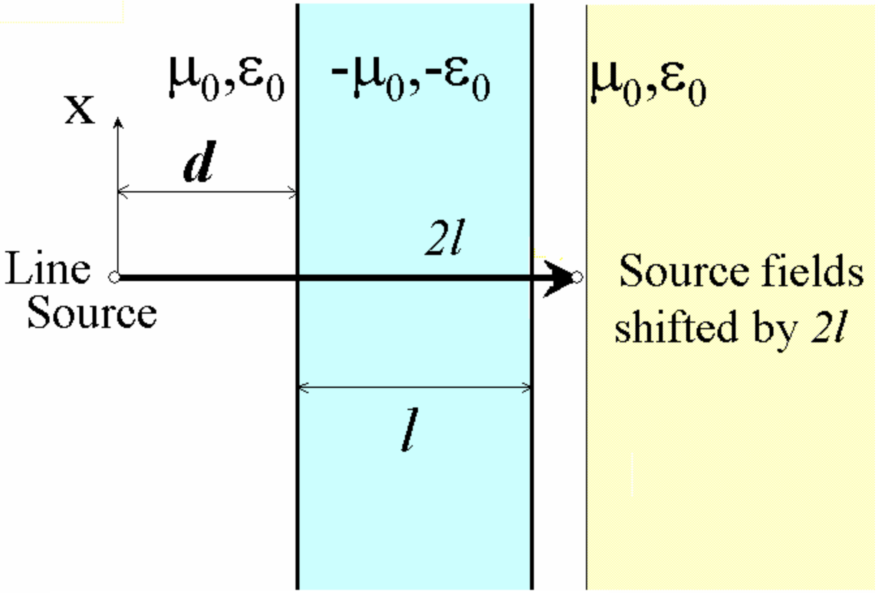
$$\varepsilon/\varepsilon_0 = \mu/\mu_0 = -1 + i\,\delta$$



$$E_x^{\mathbf{tr}}(x,z) = \lim_{\delta\to 0}\int_{-\infty}^{+\infty} T_0(k_x)\mathcal{T}_{TE}(k_x)e^{i(\gamma_0 z + k_x x)}dk_x \qquad z > d + l$$

$$\Delta Z = l - d$$

LOSSY SLAB



$$E_x^{\mathbf{tr}}(x, z) \approx \int_{-H_\delta}^{+H_\delta} T_0(h) e^{i[hx + \gamma_0 (z - 2l)]} dh$$

$$H_\delta \approx \sqrt{\left(\frac{1}{l} \ln \delta\right)^2 + \left(\frac{2\pi}{\lambda_0}\right)^2}$$

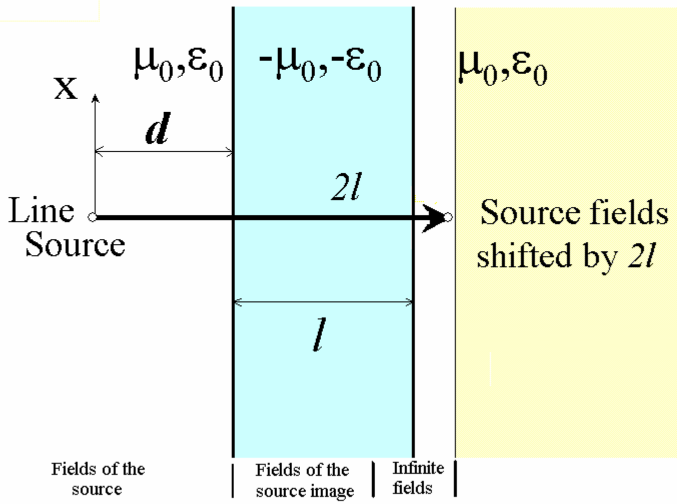
If $R_e = 5$, $l = \lambda_0$

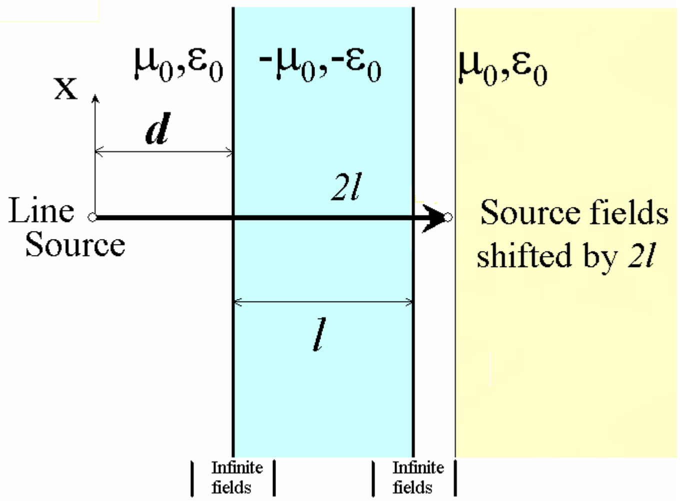
then $\delta \approx 4.3 \times 10^{-14}$

$$\Delta x = \frac{1.53\pi}{H_\delta} = \frac{.76}{R_e} \lambda_0$$

If $R_e = 2.5$, $\delta \approx 5.6 \times 10^{-7}$

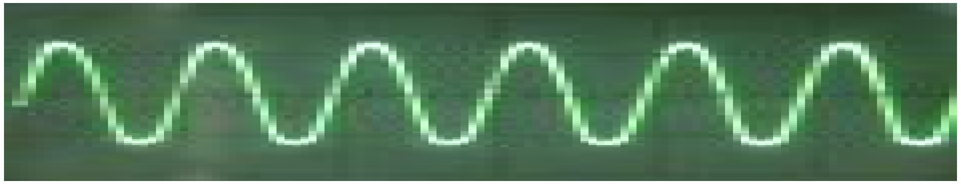
Why the infinite fields?



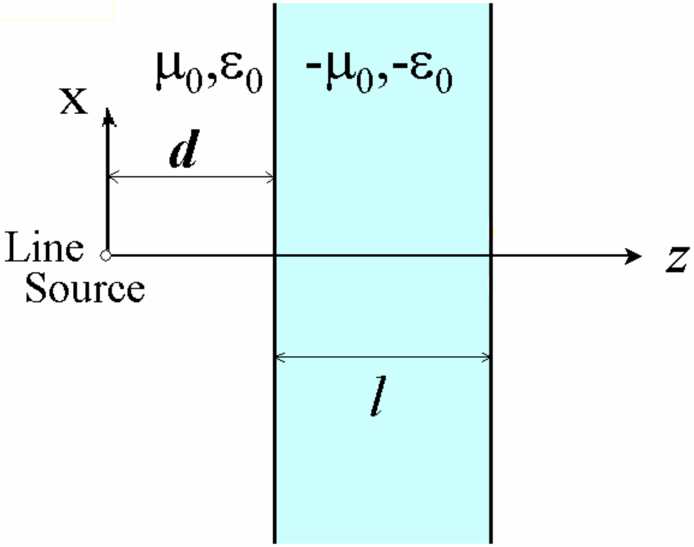


The infinite continuous spectrum of plasmon surface waves that satisfy ME's at -1 DNG interfaces carry zero total energy (have infinite Q), and thus produce infinite resonant fields about the interfaces of the -1 DNG slab. It is analogous to exciting a lossless cavity at a resonant frequency.

FINITE LENGTH SINUSOID



$t = -t_0$ $t = t_0$



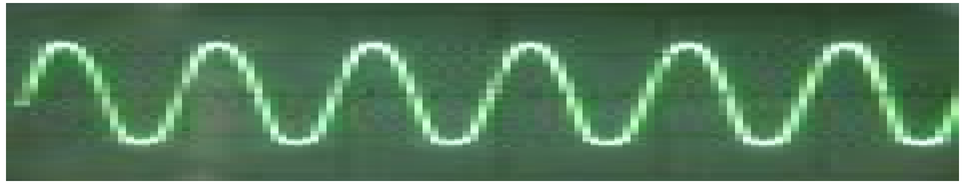
For an magnetic line current

$$T_0(k_x) \approx E_0/k_0$$

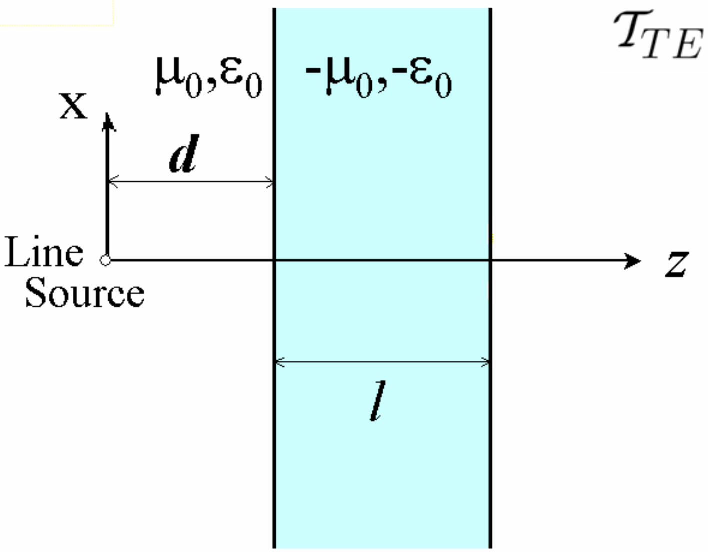
$$T(\omega) = \frac{\sin\left[(\omega - \omega_0)t_0\right]}{2\pi(\omega - \omega_0)}$$

$$\overset{+}{E}{}_x^{tr}(x,z,t) = \int_0^{\infty} T(\omega)e^{-i\omega t}d\omega \int_{-\infty}^{+\infty} T_0(k_x)\mathcal{T}_{TE}(k_x)e^{i(\gamma_0 z + k_x x)}dk_x$$

FINITE LENGTH SINUSOID



t = -t_0 t = t_0



$$\mathcal{T}_{TE}(k_x) = \frac{4e^{-i\gamma_0 l}}{\left(2 + \frac{\epsilon\gamma_0}{\epsilon_0\gamma} + \frac{\epsilon_0\gamma}{\epsilon\gamma_0}\right)e^{-i\gamma l} + \left(2 - \frac{\epsilon\gamma_0}{\epsilon_0\gamma} - \frac{\epsilon_0\gamma}{\epsilon\gamma_0}\right)e^{i\gamma l}}$$

$$\gamma_0(\omega) = (\omega^2\mu_0\epsilon_0 - k_x^2)^{\frac{1}{2}}$$

$$\gamma(\omega) = (\omega^2\mu\epsilon - k_x^2)^{\frac{1}{2}}$$

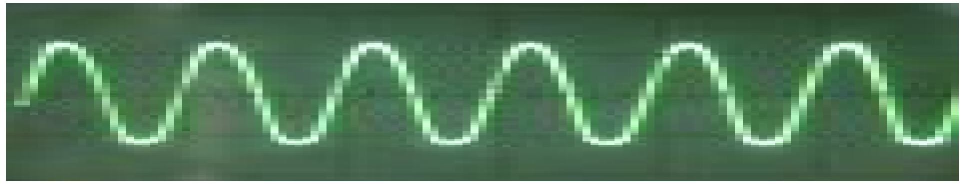
$$\kappa(\omega) = \frac{\epsilon(\omega)}{\epsilon_0} = \frac{\mu(\omega)}{\mu_0} = -1 + \boxed{\frac{4}{\omega_0}(\omega - \omega_0)} + \cdots$$

From causality ($v_g \leq c$) and energy conservation

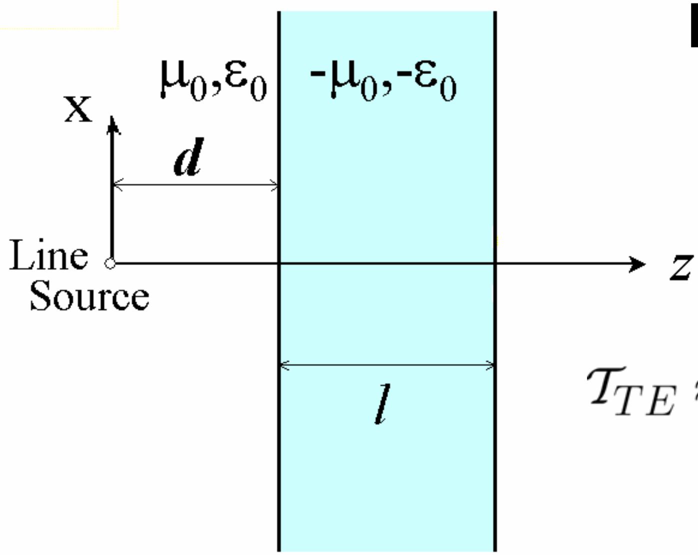
$$\boxed{\frac{d\kappa}{d\omega}(\omega_0) \geq \frac{4}{\omega_0}}$$

$$\overset{+}{E}{}_x^{\mathrm{tr}}(x,z,t) = \int_0^\infty T(\omega)e^{-i\omega t}d\omega \int_{-\infty}^{+\infty} T_0(k_x)\mathcal{T}_{TE}(k_x)e^{i(\gamma_0 z + k_x x)}dk_x$$

FINITE LENGTH SINUSOID



$t = -t_0$ $t = t_0$



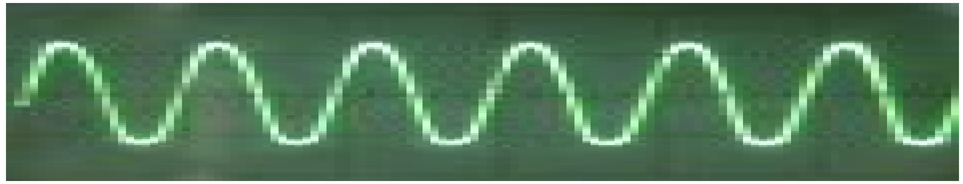
For large enough $t_{0,}$ we have

$$\mathcal{T}_{TE} \approx e^{-i\gamma_0 2l}, \qquad k_x^2 < k^2$$

$$\mathcal{T}_{TE} \approx \frac{4e^{|k_x|l}}{-[4(\omega-\omega_0)/\omega_0]^2 e^{|k_x|l} + 4e^{-|k_x|l}}, \qquad k_x^2 > k^2$$

$$E_x^{\rm tr}(x,z,t) = \int_0^\infty T(\omega)e^{-i\omega t}d\omega \int_{-\infty}^{+\infty} T_0(k_x)\mathcal{T}_{TE}(k_x)e^{i(\gamma_0 z + k_x x)}dk_x$$

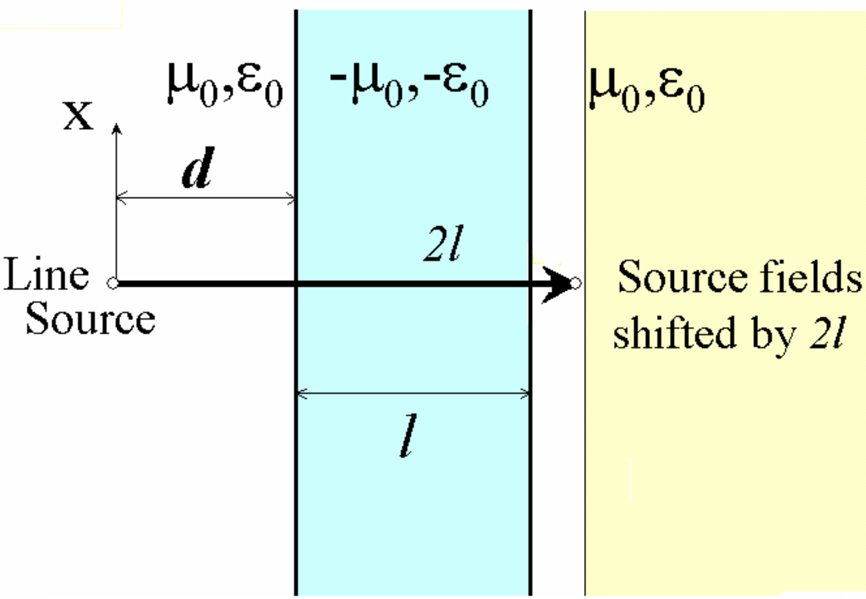
FINITE LENGTH SINUSOID



t = 0 t

Integrating over ω, and shifting the start time to t = 0, we find

$$E_x^{+\,\mathrm{tr}}(x,z,t) \approx e^{-i\omega_0 t}\int_{-K}^{+K} T_0(k_x)\, e^{i[\gamma_0(z-2l)+k_x x]}dk_x$$



$$K = \sqrt{\left[\frac{1}{l}\ln\left(f_0 t\right)\right]^2 + \left(\frac{2\pi}{\lambda_0}\right)^2}$$

$$\Delta x \approx \frac{1.53\pi}{K}$$

If $f_0 = 10^{10}$, $K = 5\, k_0$, $l = \lambda_0$

then $t \approx$ 39 **minutes**.

If $K = 2.5\, k_0$, $t \approx 2 \times 10^{-4}$ **seconds.**

NUMERICAL RESULTS

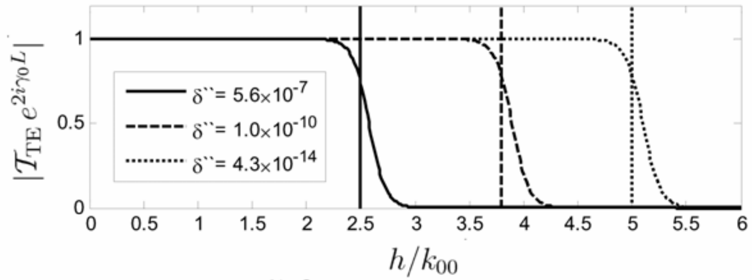


FIG. 2: The magnitude of $\mathcal{T}_{\rm TE}(h)\, e^{2i\gamma_0 L}$ for a lossy slab as a function of h/k_{00} for $\delta'' = 5.6\times 10^{-7}$, $\delta'' = 1.0\times 10^{-10}$, and $\delta'' = 4.3\times 10^{-14}$. The corresponding values for the resolution enhancements are $R_e = 2.5$, $R_e = 3.8$, and $R_e = 5$, as indicated with the vertical lines.

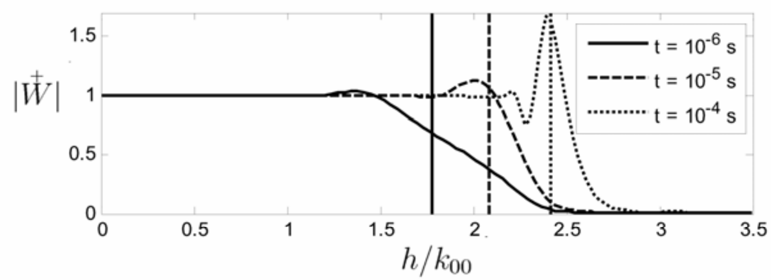


FIG. 3: The normalized magnitude of the analytic time-domain spectrum as a function of h/k_{00} for $t = 10^{-6}$ s, $t = 10^{-5}$ s, and $t = 10^{-4}$ s. The corresponding values for the resolution enhancements are $R_e = 1.8$, $R_e = 2.1$, and $R_e = 2.5$, as indicated with the vertical lines.

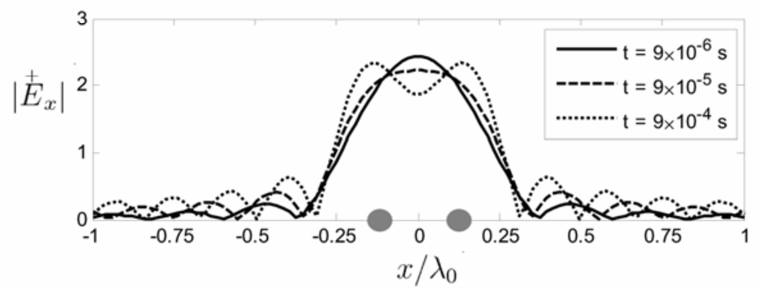
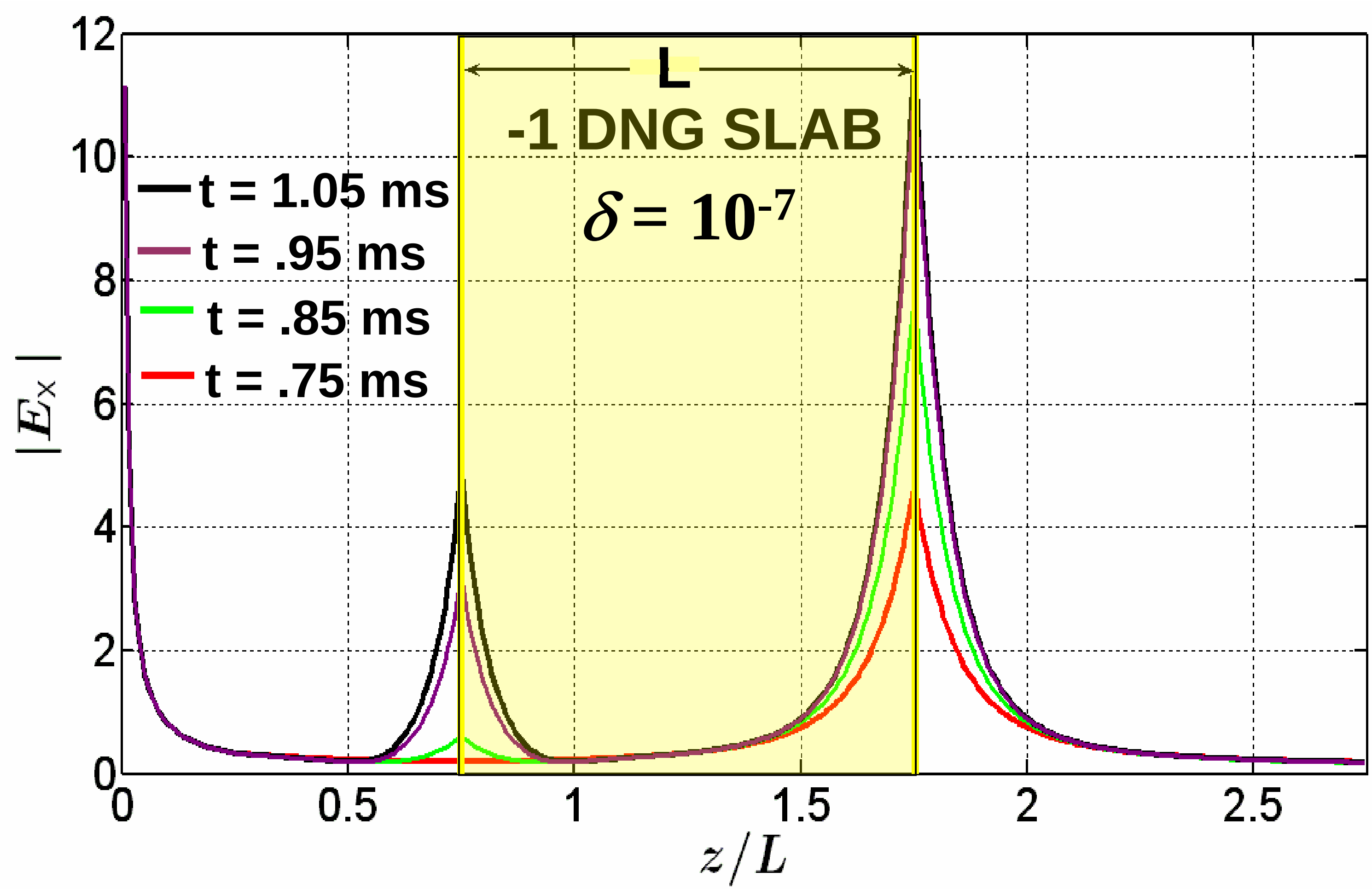


FIG. 4: The normalized magnitude of the analytic time-domain electric field from two line sources as a function of x/λ_0 for $t = 9\times 10^{-6}$ s, $t = 9\times 10^{-5}$ s, and $t = 9\times 10^{-4}$ s, corresponding to a resolution of $\Delta x = 0.37\lambda_0$, $\Delta x = 0.32\lambda_0$, and $\Delta x = 0.28\lambda_0$, respectively. The line sources are located at $(x, z) = (\pm\lambda_0/8, 0)$ as indicated by the two gray dots.

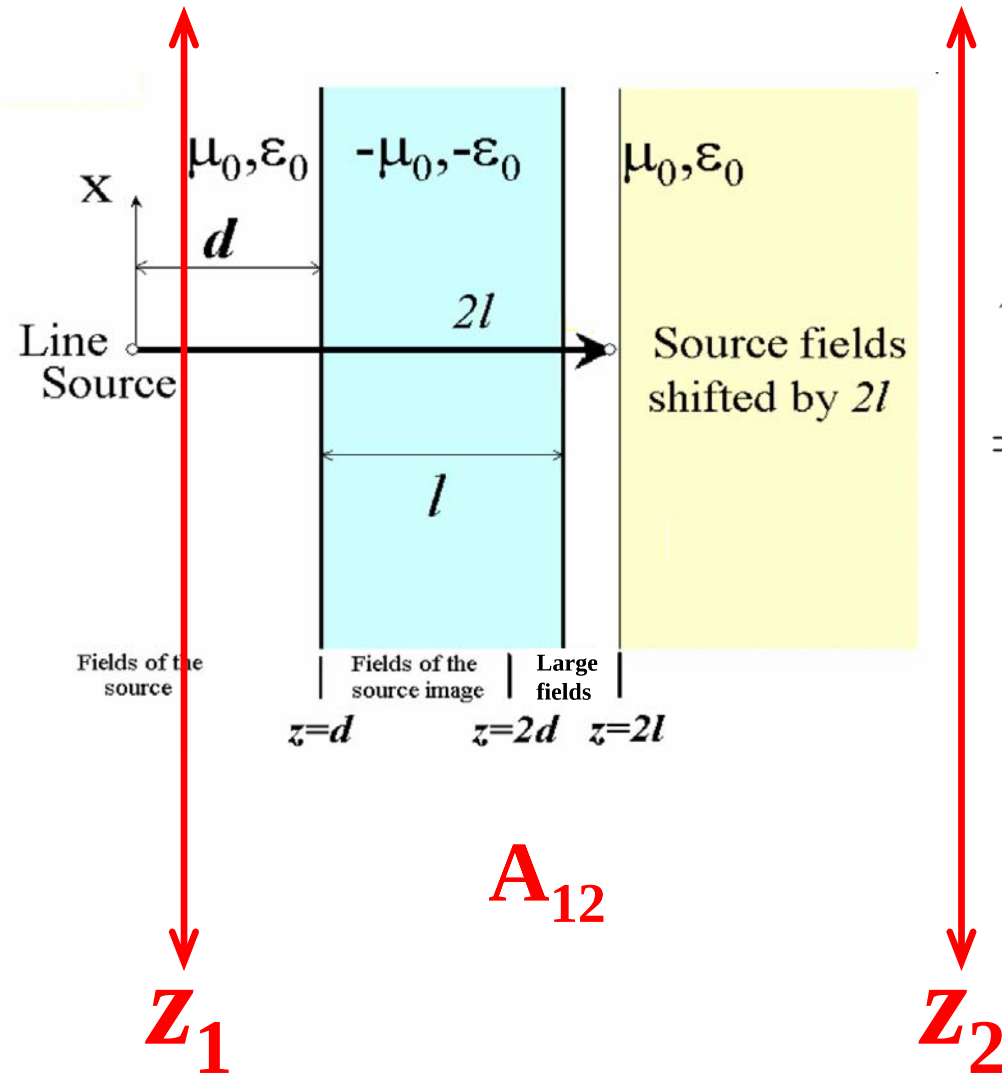
CENTER-LINE FIELD VS TIME IN LOSSY DNG SLAB (LINE SOURCE AT Z=0)



$$\overset{+}{E}_x(x,z,t) \overset{t\to\infty}{\approx} \frac{E_0\, e^{-i\omega_0 t}}{\pi k_{00}\sqrt{x^2+(2L-z)^2}}\, \tau^{2-z/L} \cos\left(\frac{x}{L}\ln\tau - \tan^{-1}\frac{x}{2L-z}\right)$$

$$(\tau = f_0 t)$$

FIELDS OBEY POYNTING'S THEOREM



$$\int_{z_1 - z_2} \hat{\mathbf{z}} \cdot (\mathbf{E} \times \mathbf{H})\, dx$$

$$= \frac{1}{2}\frac{d}{dt} \int_{A_{12}} \left[\epsilon(\mathbf{r})|\mathbf{E}|^2 + \mu(\mathbf{r})|\mathbf{H}|^2 \right] dA$$

SUMMARY

- The TD solution to the -1 DNG slab (coupled with bounds on frequency variation of μ and ε) reveals the evolution of finite EM fields.
- The TD solution shows that the -1 DNG slab does not violate Maxwell's equations or energy conservation.
- Unbounded reactive fields in lossless slab as source duration time approaches infinity is analogous to the unbounded reactive fields in lossless cavity resonators.
- Losses dictate that significant enhancement of source resolution can occur only if both the source distance and slab thickness are small fractions of a wavelength.
- Since the source must be so close to the slab and the slab does not focus the fields, its utility as a lens is questionable.

RESURGENCE IN ELECTROMAGNETICS

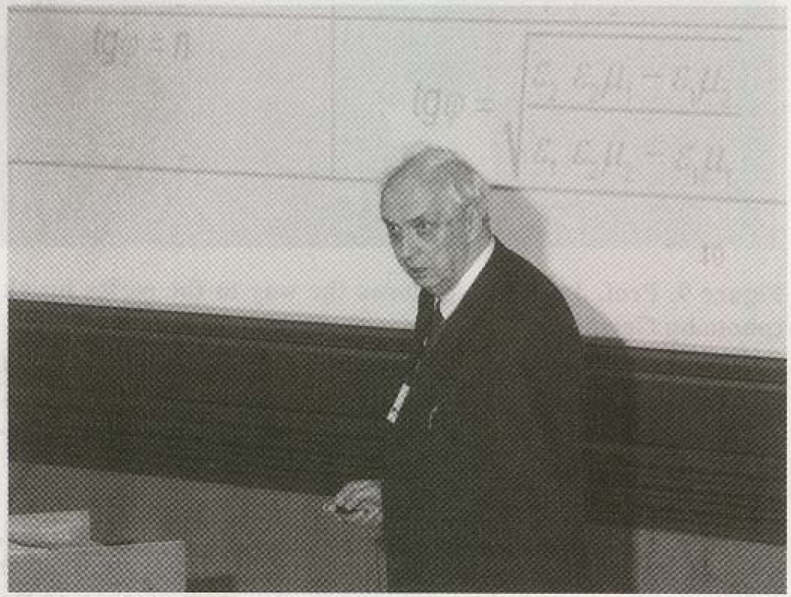


Figure 2. Prof. Victor Veselago, who introduced the concept of negative refraction (photo by Zürcher).

Figure 3. Prof. John Pendry discusses Swiss Roll structures (photo by Zürcher).

IEEE Antennas and Propagation Magazine, Vol. 47, No. 3, June 2005