MVO: Origins and Current Problems

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- ...and a specified risk over this same period.
- These two quantities are balanced against each other in some ‘optimal’ way.

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The above is generally referred to as mean-variance optimization. In terms of the original discussion, we are equating

- Reward with expected return
- Risk with portfolio variance
- We are minimizing risk subject to a minimum expected return

This formulation is normative and not empirical. Financial data are nonstationary. Additionally, measurements have error.
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In time series, we often think of observations through time as random variables.

Generally, in time series, we want to have the joint distribution of time lagged variables to only depend on the lag - not the time.

For example, we might expect that the correlation between two stocks’ returns might be the same over distinct 200 day periods. Or, a covariance matrix might be assumed to remain the same in the future.

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The question arises...
Can we test if financial time series are weakly stationary?
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![Trailing 200 Day Mean Return: SPX](chart.png)
Putting aside practical questions for a moment, there are interesting features of the problem

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like, what happens when we solve the problem for various \( \alpha \)'s?
We get something called the efficient frontier.
Now, what if there is a risk free asset. One with no volatility. We get the ‘market portfolio’ and $\beta$

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If we put on our normative hats, we start making claims:

- You’d always want a portfolio on the efficient frontier
- If there is a risk free asset, you’d always want to be on the market line
- Ergo, all risk is systemic and you just need to figure out how much of the market portfolio you want
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We also obtain another optimization problem, namely, finding the optimal Sharpe ratio:

$$\max_{\{w \in \mathcal{W}\}} \frac{\mu^T w - r_f}{\sqrt{w^T \Sigma w}}$$

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The results are neat.
You have to ask yourself, though, what made such an oversimplified statement possible?
Everything in the setting above, the Capital Asset Pricing Model, assumes that means are stable through time. And so are covariances.
The good news for the future practitioner: they’re not. At all.
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The good news for the future practitioner: they’re not. At all.
A disclaimer: this view on the origins of modern portfolio theory may not be the norm. In fact, Markowitz was quoted as saying “Diversifying sufficiently among uncorrelated risks can reduce portfolio risk toward zero” as recently as 2008. However,
And in case that doesn’t seem fair:
There are times when “correlation goes to one.” Hence true diversification may fail exactly when you need it most.

We see the need to mitigate definable risks according to our understanding (model).

We see here the inherit importance of understanding our underlying; e.g., beginning with the premise that our data is nonstationary, or understanding what influences stock prices, say.

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There is no right way to ‘mitigate definable risks according to our understanding’.
Again, there are tools, not answers. Tutuncu suggests a robust optimization procedure using interval sets around expectation and the elements of the covariance matrix. Goldfarb and Iyengar suggest that you define a linear factor model and use elliptical uncertainty sets based on regression coefficients.
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Both of these options have their place. In the case of Tutuncu, the practitioner has some added flexibility, but may lose structure. The negation of this statement is true for Goldfarb and Iyengar. Below, we create a hybrid of the two, using a practitioner informed elliptical uncertainty set based on a linear factor model. We are therefore motivated to construct some linear factor model for returns.
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This linear factor model is an example of attempting to ‘understand our underlying’.
One way was established by the work of Fama and French. They suggested that risk in equities and bonds could be explained in (large) part by

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Let’s ignore the EMH for a moment, and look at a cross-sectional regression model with factors:

- Size: market cap
- Value: book to price
- Leverage: debt to equity
- Asset growth: measured year over year
- Momentum and mean reversion: one month trailing returns
- We also include an intercept term
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If $r$ is a forecast, and $V$ is a design matrix, we have

$$r = V^T \cdot f + \epsilon$$

The vector $f$ is determined here by regression, and so is the assumed distribution of $\epsilon$. Below we assume $\epsilon \sim N(0, D)$, with $D$ diagonal.
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G/I suggest the robust problem based on temporal regression

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\max_{\{w \in W\}} \min_{V \in S_v, \mu \in S_m, D \in S_d} \frac{\mu^T w - r_f}{\sqrt{w^T \Sigma w}}.
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with

- \(S_v = \{V : V = V_0 + W, \|W_i\|_g \leq \rho_i, i = 1, \ldots, n\}\)
- \(S_m = \{\mu : \mu = \mu_0 + \xi, |\xi_i| \leq \gamma_i, i = 1, \ldots, n\}\)
- \(S_d = \{D : D = \text{diag}(d), d_i \in [d_i, d_i], i = 1, \ldots, n\}\)

Here, \(W_i\) denotes the \(i\)th column of \(W\) and \(\|w\|_g = \sqrt{w^T G w} \).
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As noted earlier, G/I determine their robust model based on regression. In particular, G, γ, and ρ are based on confidence intervals and (temporal) regression coefficients. The resulting formula may be written (and solved) as a second order cone program. We alter the construction of G, γ, and ρ as we are interested in cross-sectional regressions.
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We consider

\[ B = [f_{t-p} \cdots f_{t-2} f_{t-1}] . \]

\[ G = \left( B \cdot B^T - (B \cdot 1) \cdot (B \cdot 1)^T \right) \]

and \( \gamma_i = \sqrt{(B \cdot B^T)^{-1}}_{(1,1)} c_{p,m}(\omega) \cdot \sigma^2_i, \rho_i = \sqrt{c_{p,m}(\omega) \cdot \sigma^2_i} \)

We have assumed that \( f(1) \) is the coefficient related to the intercept of our model.

We also modify the expected return vector from their formulation.
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Using the above uncertainty sets, and a 95% confidence interval, we obtain robust portfolios seeking to maximize the Sharpe ratio.

There are several details to keep in mind when performing backtests, such as,

- awareness of survivorship bias,
- resisting the temptation to snoop the data,
- and a concerted effort must be made to introduce ‘future’ information.

I took all of this into consideration and attempted to not introduce any of the above failings in the results that follow.
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Below we include results for three problems:

- Maximizing the Sharpe ratio using the trailing mean
- Maximizing the Sharpe ratio using a forecasted return based on the model above
- Maximizing the Sharpe ratio using the robust counterpart cited and the factor model

Our universe consists of large cap stocks - those with market cap over $10 billion at the time of portfolio formation. We assume a one month holding period below.
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<table>
<thead>
<tr>
<th>Annual Return (%)</th>
<th>S&amp;P 500</th>
<th>Nominal Sharpe μ: Trailing Mean</th>
<th>Nominal Sharpe μ: forecast</th>
<th>Robust Sharpe μ: forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002 (beginning March)</td>
<td>-17.15</td>
<td>0.33</td>
<td>-0.34</td>
<td>0.21</td>
</tr>
<tr>
<td>2003</td>
<td>13.02</td>
<td>5.20</td>
<td>4.92</td>
<td>2.10</td>
</tr>
<tr>
<td>2004</td>
<td>10.93</td>
<td>1.03</td>
<td>2.64</td>
<td>11.01</td>
</tr>
<tr>
<td>2005</td>
<td>6.45</td>
<td>24.94</td>
<td>5.62</td>
<td>7.30</td>
</tr>
<tr>
<td>2006</td>
<td>12.10</td>
<td>-10.69</td>
<td>-0.27</td>
<td>4.55</td>
</tr>
<tr>
<td>2007</td>
<td>5.75</td>
<td>57.13</td>
<td>21.78</td>
<td>-0.49</td>
</tr>
<tr>
<td>2008</td>
<td>-39.49</td>
<td>2.49</td>
<td>3.36</td>
<td>8.78</td>
</tr>
<tr>
<td>2009 (through June)</td>
<td>2.56</td>
<td>-26.52</td>
<td>18.38</td>
<td>-1.21</td>
</tr>
<tr>
<td>Annualized Return (%)</td>
<td>-2.78</td>
<td>4.78</td>
<td>7.37</td>
<td>4.31</td>
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<tr>
<td>Annualized Volatility</td>
<td>15.78</td>
<td>24.28</td>
<td>15.93</td>
<td>6.19</td>
</tr>
<tr>
<td>Annualized Sharpe Ratio</td>
<td>(0.10)</td>
<td>0.32</td>
<td>0.53</td>
<td>0.71</td>
</tr>
<tr>
<td>Max Gain</td>
<td>9.39</td>
<td>18.86</td>
<td>15.85</td>
<td>6.82</td>
</tr>
<tr>
<td>Max Drawdown</td>
<td>-16.94</td>
<td>-26.69</td>
<td>-14.36</td>
<td>-4.85</td>
</tr>
<tr>
<td>S&amp;P Relative</td>
<td>β</td>
<td>1.00</td>
<td>(0.11)</td>
<td>(0.09)</td>
</tr>
<tr>
<td></td>
<td>α</td>
<td>0.00</td>
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<td>0.69</td>
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chris bemis  Whitebox Advisors  MVO: Origins and Current Problems
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- Addressing the nonstationarity of underlying returns, *even in this simple model*, produces superior results to assuming the past is like the future; viz., compare nominal problems above.

- Addressing model uncertainty provides a significant tool to enhance performance by various metrics; viz., compare nominal with forecast to robust.

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I would like to thank the IMA for holding their Mathematical Modeling in Industry Workshops. Much of the data shown in the above table was calculated by students from the 2009 workshop (XIII).
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