



Modeling Portfolio Credit Risk

William Morokoff
New Product Research
Moody's KMV

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Outline

- **Credit Risk Overview**
- **Brief History of Quantitative Credit Modeling**
- **Risk Modeling for Credit Portfolios**
- **Monte Carlo Simulation of Credit Portfolios**

Credit Risk

- **The risk that a counterparty in a financial contract may not pay obligations – i.e. the risk of default.**
- **Recent examples: Enron, K-Mart, United Airlines**
- **Affects corporate bonds and loans and associated derivatives, plus some governmental bonds.**
- **Time frame for risk usually in years – what is the probability of default over the next year?**
- **Other credit risky instruments: Asset Backed Securities (ABS) based on credit cards, mortgages, student loans, aircraft leasing, etc.**

Modeling Tools Used in Credit Risk

- **Empirical Data Handling Methods:**
Model Specification, Model Calibration
- **Statistics:** Regression, Hypothesis Testing, Model Validation
- **Stochastic Processes**
- **Numerical Methods:** Monte Carlo
- **Applied Math:** Multivariate Probability, Linear Algebra, Fourier Transforms, etc.

People Interested In Credit Risk

- **Bankers – Portfolio and Risk Managers, Lenders**
- **Asset Managers, Insurance Companies, Hedge Funds, Corporations with Trade Credit**
- **Debt Structures**
- **Regulators and Rating Agencies**
- **Finance Researchers – Corporate Finance**
- **Economists**
- **Technology Developers**
- **Quants**

Simple Corporate Finance Model

Corporations finance projects by through

- **Equity (stock):** ownership of part of the firm
- **Debt:** borrowing with obligation to repay

Firm Value = Market Value of Equity + Market Value of Debt

Default occurs when

Firm Value < Liabilities

Structural Model for Default Probability

- **Default Probability can be inferred from**

A = Firm Value

D = Liabilities

σ = Firm Value Volatility

- **DD = Distance to Default**

$$DD = \log(A/D) / \sigma$$

Number of standard deviations away from default

- **DD is good measure for ranking firms. Need empirical default data to get default probabilities.**

Modeling Questions for Structural Models

- **How can firm value be determined? – not observable in the markets**
- **How do firm value, liabilities, volatility evolve over time?**
- **How are they correlated with other companies?**
- **How does default probability determine market price of a bond, loan, etc.? Other factors include:**
 - **Recovery**
 - **Market Price of Risk**
 - **Liquidity**
 - **Size Premium/ Rating effect**
 - **Option features of instrument**

Selective Highlights of Quantitative Credit Modeling History

- **Merton Model (1973-74) – Black and Scholes and Merton developed framework based on observation that Equity has a call option on the firm value. Much academic work follows, but difficulties in getting default probabilities from models.**
- **Banks use scoring or internal ratings models; Altman's Z score adds econometric insight.**
- **Rating Agencies provide qualitative assessment.**
- **Vasicek-Kealhofer model (1989) – Structural model in Merton framework calibrated to default data; first widely successful structural model.**

Selective Highlights of Quantitative Credit Modeling History

- Portfolio credit models combining default probabilities and correlations become more widely used by mid 1990s.
- Jarrow-Turnbull (1995) – Introduction of “Reduced Form” models, describing stochastic process for evolution of credit risk in terms of default intensities as a component of yield spread. Duffie-Singleton (1999) further develop the idea to give the most commonly used reduced form model.
- Private firm default models develop in late 1990s.
- Today: Many models for default probabilities, correlations, portfolio modeling, and credit derivatives.

Risk Modeling for Credit Portfolios

ω = \$ exposure size

$V(x, t)$ = Value of loan in credit state x
at time t relative to par

$\Pi(t)$ = Value of portfolio

$$\Pi(t) = \sum_{i=1}^N \omega_i V_i(x_i, t)$$

(x_1, \dots, x_N) correlated random variables

Risk Modeling for Credit Portfolios

$$L(t) = \text{Loss}$$

$$L(t) = E(\Pi(t) | \text{no defaults}) - \Pi(t)$$

$$EL = \text{Expected Loss}$$

$$UL = \text{Unexpected Loss} = \sigma(\Pi)$$

$$C = \text{Capital} \sim \text{Value at Risk}$$

$$RC_i = \partial UL / \partial \omega_i = \text{Risk Contribution}$$

$$UL = \sum_{i=1}^N \omega_i RC_i$$

Simple Homogenous Default/No-Default Model

- Assume all loans have same exposure, default probability, correlation, and recovery

$$L(t) = \omega \cdot \text{LGD} \cdot \sum_{i=1}^N \chi(x_i)$$

x_i iid, uniform on $[0, 1]$

$$\chi(x) = \begin{cases} 1 & \text{if } x \leq EDF \\ 0 & \text{if } x > EDF \end{cases}$$

Simple Homogenous Default/No-Default Model

- Easy to compute EL and UL as functions of EDF, LGD and correlation (joint default frequency JDF)
- Question: How much diversity can be achieved for a given EDF and correlation? How many loans do you need before the portfolio is well diversified?
- Probability distribution of loss is not obvious because of correlations. For single factor models, Semi-Analytic methods allow the distribution to be expressed as a 1 dimensional integral.

Monte Carlo Simulation for Credit Portfolios

- **Single Horizon Analysis:**

- Draw random, correlated credit states at horizon for all obligors
- Value each facility at the horizon as a function of its credit state
- Compute portfolio value/loss at horizon
- Repeat many times to build probability distribution of losses and estimate associated statistics such as risk contribution.

Monte Carlo Simulation for Credit Portfolios

- **Computational Problem:** Capital is often required at the 10 bp level (i.e., 99.9% VaR). This requires 100,000 simulation runs for reasonable accuracy; very slow for large portfolios. Accurate Tail Risk Contribution requires even more runs.
- **Solution:** Importance Sampling. Put more sample points in loss tail and weight samples non-uniformly.
- **Difficulties:** Identifying ways to generate large (but not too large) loss sample points; making sure the rest of the distribution is still adequately sampled.

Monte Carlo Simulation for Credit Portfolios

- **Importance Sampling usually depends on tailoring sampling to function (here the portfolio).**
- **Credit portfolio property: Tail of loss distribution created by multiple correlated defaults. Increasing correlations will lead to more large loss events.**
- **If firm value returns are assumed Gaussian, default correlations can be increased by increasing value return correlations.**