



# **How to find Surface Ships and Small Go-Fast Craft Wakes in the Ocean**

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# Talk Objectives:



- ***Describe the kinds of projects on which we work***
- ***Describe the surface ship wake detection problem***
- ***Describe the open mathematical issues, and get some leads (caveat)***

# Where our group fits in Lockheed:



<b><i>Lockheed Martin</i></b>	<b><i>\$25 Billion in Sales</i></b>	<b><i>140,000 employees</i></b>
<b><i>Systems Integration (1 of 4 divisions)</i></b>	<b><i>\$9.6 Billion in Sales</i></b>	<b><i>31,000 employees</i></b>
<b><i>Tactical Systems (1 of 13 companies)</i></b>		<b><i>1,300 employees</i></b>
<b><i>Maritime Surveillance Aircraft (1 of 4 business units)</i></b>		<b><i>300 employees</i></b>
<b><i>Advanced Sensors and Algorithms Group</i></b>		<b><i>10 employees</i></b>



# **Problem Areas on which our group focuses**

- ***Finite Set Statistics: Unified Framework for Data Fusion and multi-target tracking/ID***
- ***Fine grain tracking in high scintillation environments***
- ***Data Fusion Performance Measuring***
- ***Line Tracking***
- ***Target Detection and Tracking in high clutter environments***
- ***Passive Surveillance Systems***
- ***Automatic Target Recognition***
  - ***SAR (Synthetic Aperture Radar)***
  - ***HRR (High Resolution Radar)***
  - ***Fusing of disparate Anti-Submarine Warfare Sensors***

# How to detect anything (wakes)



- ***Characterize Signal and its motion***
- ***Characterize Environmental Noise***
- ***Characterize Sensor Noise***
- ***Apply Filtering Techniques***

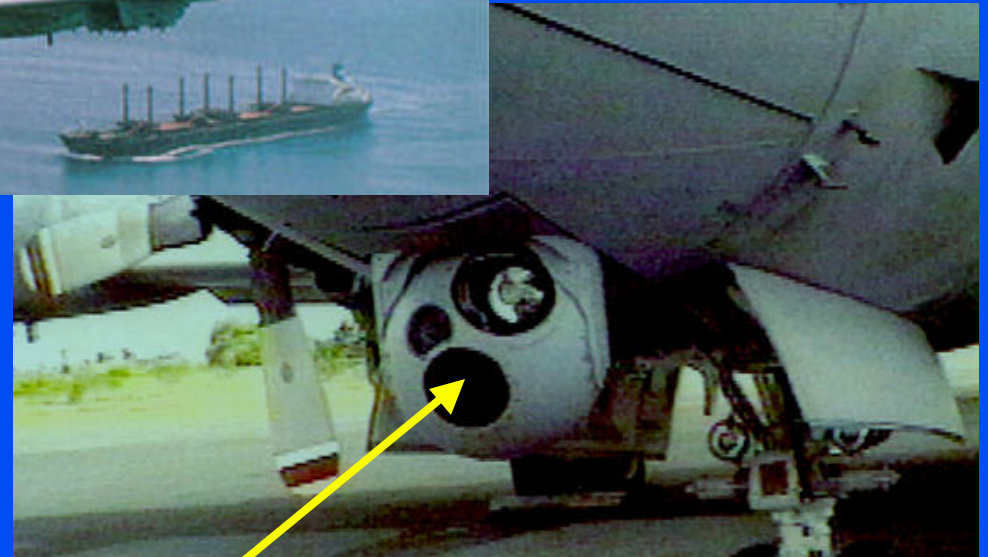
## ***Assumptions/Goals***

- ***Assume extremely low SNR.***
- ***Maximize altitude.***
- ***Minimize magnification.***
- ***Quantify PD and PFA.***

# Sensor on the P-3



- ***Use AIMS sensor.***



***AIMS (EO/IR) Sensor***

# Characterize Signal



- **The signal is a solution to a Free Boundary Value Problem**
  - **$\Phi$  is the velocity potential**
  - **$\eta(x,z;t)$  is the height of the free surface at  $(x,z)$  at time  $t$**

$$\nabla^2 \Phi = \Phi_{xx} + \Phi_{yy} + \Phi_{zz} = 0$$

$$\Phi_x h_x - \Phi_y + \Phi_z h_z + h_t = 0 \quad \text{for } y = h(x, z; t)$$

$$g h + \Phi_t + \frac{1}{2} (\Phi_x^2 + \Phi_y^2 + \Phi_z^2) = F(x, z; t) \quad \text{on } y = h(x, z; t)$$

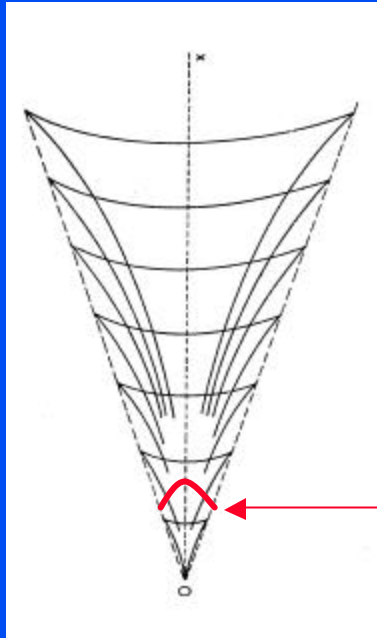
with  $F = 0$  except at disturbance



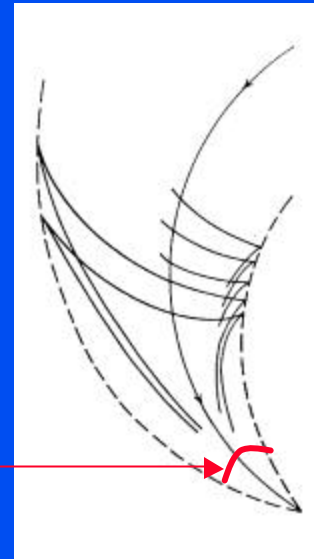
# Some Theoretical Results

*We will assume constant speed, constant turn radius.*

*We will assume hull does not affect wake structure.*



39 Degrees



*Structure scales linearly in speed.*

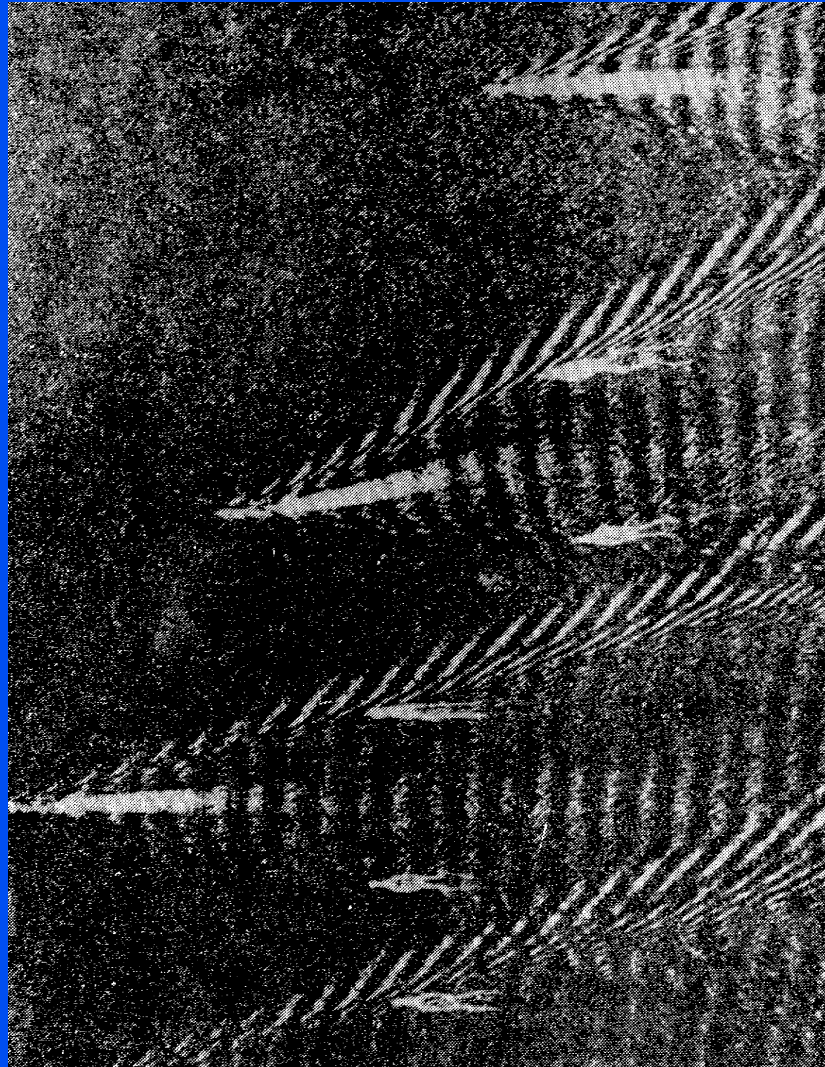
*Angle, and basic shape fixed for different speeds.*

*Analytic Results appear to be intractable.*

# Examples of Wakes:



# Examples of Wakes:



# Partial Numerical Results

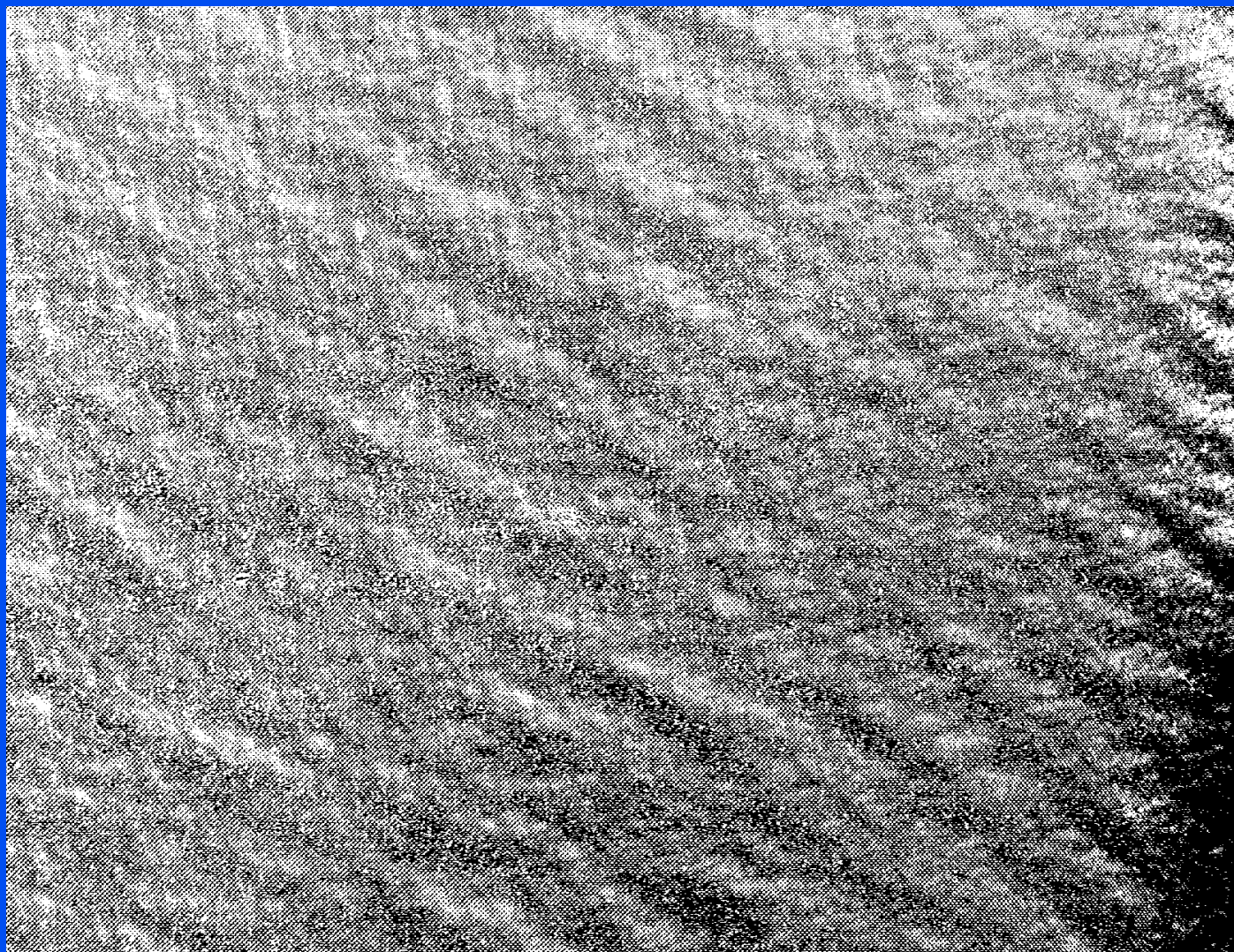


# Characterize Ocean Background



- ***Current Assumption: gaussian, independent pixel to pixel.***
- ***Sources of background clutter:***
  - ***Wind***
  - ***Shore***
  - ***Shipping***
- ***Buoy data exists (could be applied to single pixel model)***
- ***I have not yet found 2D data models applicable to our problem***

# Example of Noise



# Characterize IR image model



- ***Speckle (Sun is a coherent IR light source)***
  - ***Filters exist to smooth speckled imagery***
- ***Currently no other modeling being performed to model the imagery***

# Characterize Sensor Noise



- ***All models are currently heuristic, but are sufficient***
- ***Sources of noise:***
  - ***Scan lines***
  - ***Heat from the electronics and optics***
  - ***Electronics Noise***
  - ***Faulty Pixels***
    - ***Slow***
    - ***Bad***

# Filtering



- ***Given:***
  - ***Observation (Signature) model***
  - ***Motion Model for the target (constant speed, constant curvature turns)***
  - ***Background noise model***
- ***We can mathematically define the optimal filtering technique, Non-linear filtering***

# Non Linear Filtering



$X_k$  = Position, speed, and curvature of target at time  $k$

$W(X_k)$  = Wave generated by target  $X_k$

$V_k$  = Ocean background at time  $k$

$Z_k$  = Observation at time  $k$

$X_{k+1} = g(X_k)$

$Z_k \approx V_k + W(X_k)$

Want to estimate the probability density function

$$f(X_k | Z_k, Z_{k-1}, \dots, Z_0)$$

If  $X_k$  markov, and  $V_k$  independent then

$$f(X_k | Z_k, Z_{k-1}, \dots, Z_0) = \frac{f(Z_k | X_k) f(X_k | Z_{k-1}, \dots, Z_0)}{f(Z_k | Z_{k-1}, \dots, Z_0)}$$

and  $f(X_k | Z_{k-1}, \dots, Z_0) = \int f(X_k | x) f(x | Z_{k-1}, \dots, Z_0) d\mathbf{m}_{X_{k-1}}$



# Particle Method Approach to Non-linear Filtering

*(1) Create  $N$  particles, distribute them uniformly.*

*(2) Have each particle move as motion model.*

*(3) Take observation.*

*randomly kill particles that don't support the hypothesis that target is in particle's location.*

*randomly duplicate particles that do support hypothesis, keeping track of ancestors.*

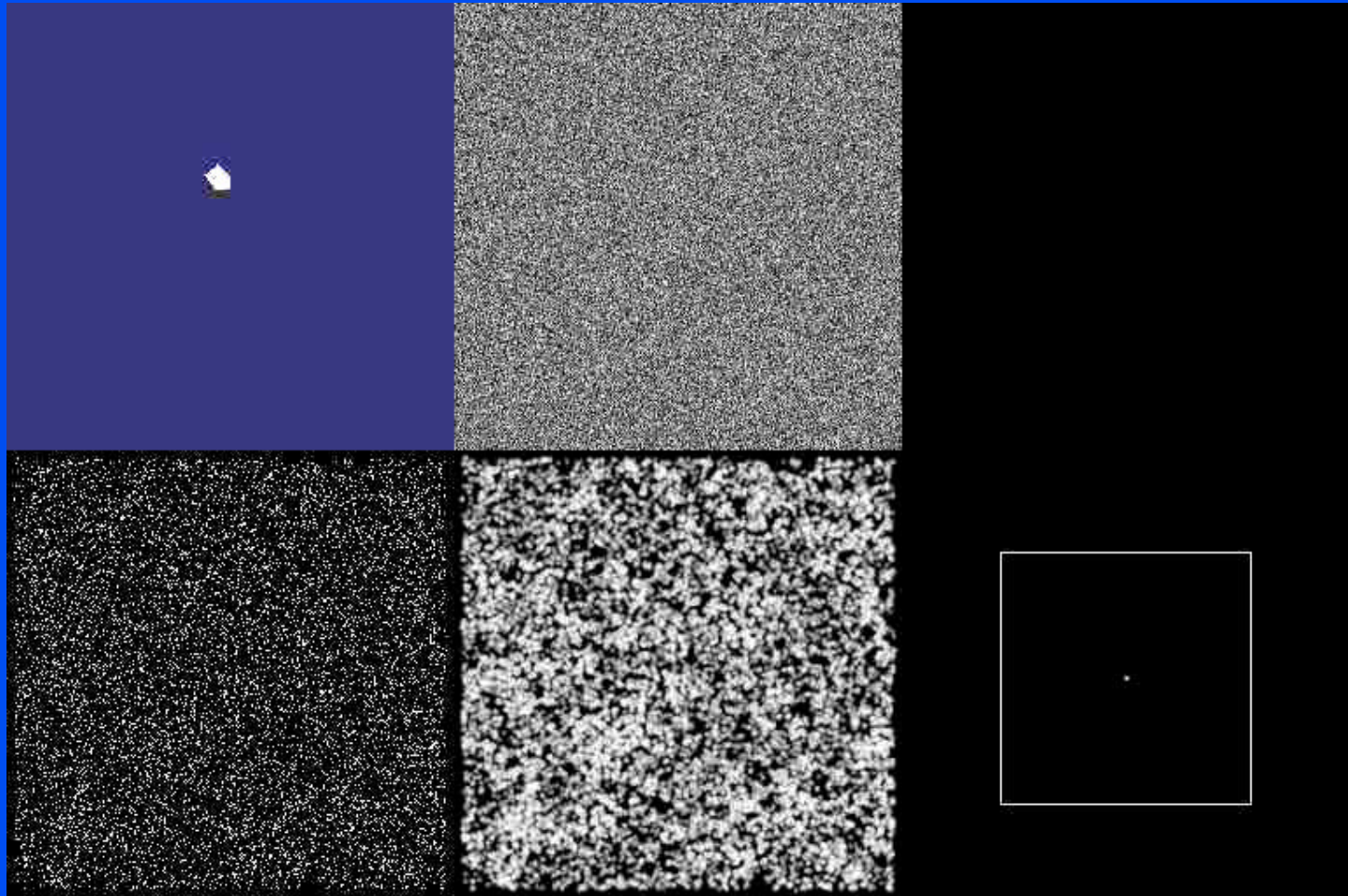
*(4) go to (2)*

*(Kouritzin)*

# Particle Movements



## *Visualization of Algorithm*



# Open Problem Summary



- ***Can we use first principles to obtain a more detailed wake signature model?***
  - ***Analytically?***
  - ***Through application of known numerical techniques?***
- ***Given a realistic wake signature, is there a transform analogous to the Hough transform for line detections in clutter?***
- ***Are there “usable” ocean surface models?***
- ***Can we characterize the dependence in time of the ocean surface?***

# Bibliography



- **Info about Lockheed:** <http://www.lockheedmartin.com>
- **Info about the P3**  
<http://www.chinfo.navy.mil/navpalib/factfile/aircraft/air-p3.html>
- **Stoker, Water Waves, Interscience Publishers, New York, 1957**
- **Johnson, A Modern Introduction to the Mathematical Theory of Water Waves, Cambridge Texts in Applied mathematics, 1997**
- **Water Simulations:** <http://graphics.stanford.edu/~fedkiw/>
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- **Ballantyne, David J.; Chan, Hubert Y.; Kouritzin, Michael A. Novel branching particle method for tracking Proc. SPIE Vol. 4048, p. 277-287, Signal and Data Processing of Small Targets 2000, Oliver E. Drummond; Ed.**

# Mathematical Technologies we use



- ***FISST***
- ***Fuzzy Logic***
- ***Multi-Hypothesis Tracking***
- ***Wavelets***
- ***PDE's ODE's SPDE's***
- ***Nonlinear-Filtering***