Ben-Gurion University of the Negev
Faculty of Engineering Science
The Department of Industrial Engineering and Management

Real Time Multi-Scale Geometric Segmentation of 3D Images

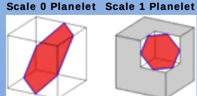
Levi Ofer, Nir Naor, Inbal Asher, Yoash Chassidim

3D Planelets (Donoho and Levi)

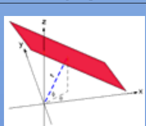
The Planelet Transform

The Planelets Transform is a Discrete Multi-Scale 3D Radon Transform applied on a dyadic cubes structure

Planelets are a set of $O(n^3\log(n))$ plane patches in a variety of locations, orientations and scales, which can be used to effectively approximate smooth surfaces in a 3D image



The 3D Radon Transform



Plane Parameters

$f(x,y,z), \quad (x,y,z)\in\Re^3$

$Rf(P)=\int_P f(s)ds$

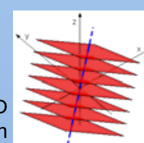
$R:\Re^3\to\Re^3$

$$P=\{(\cos(\phi)\cos(\theta)x+\cos(\phi)\sin(\theta)y+\sin(\phi)z=r):\phi\in[0,\Pi),\ \theta\in[0,2\Pi)\}$$

$$Rf(r,\phi,\theta)=\iiint f(x,y,z)\delta(r-\sin(\phi)\cos(\theta)x-\sin(\phi)\sin(\theta)y-\cos(\phi)z)dxdydz$$

Projection-Slice Theorem

$$R(r,\phi,\theta)=F_1^{-1}F_3 f(x,y,z)$$
$$f(x,y,z)=F_3^{-1}F_1R(r,\phi,\theta)$$



Projections and Slice

This theorem is the key to the 3D Fast Slant-Stack Transform, which uses a Pseudo-Spherical grid to compute a Discrete 3D Radon transform

Pseudo-Spherical (PS) Grid

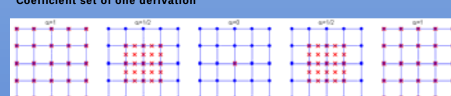
- Concentric boxes instead of Spheres.
- Uniformly spaced slopes instead of angles.
- Allows to compute the PSFT in $O(n^3\log n)$ instead of $O(n^6)$ in direct computation.

PSFFT Algorithm (Averbuch et.al.)

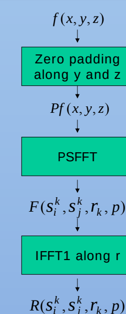
Apply 1D FFT's along x.
Apply Fractional FFT's along y with varying Alpha.
Apply Fractional FFT's along z with varying Alpha.

$$\tilde{f}_\alpha(k)=\sum_{m=-n/2}^{n/2-1} f(m)\cdot\exp\left(-i2\Pi\alpha\frac{km}{n}\right),\ k=1,\dots,n$$

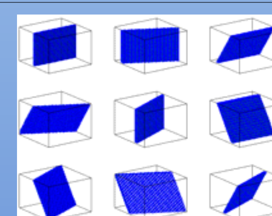
Coefficient set of one derivation



The 3D Fast Slant-Stack Algorithm



3D FSS Back-Projections

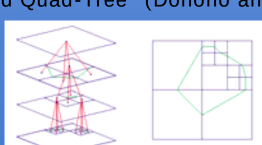


The Planelet Transform is a multi-scale 3D FSS
It also have an overall complexity of $O(n^3\log(n))$.

3D Geometric Segmentation using Planelets

Planelet Decorated Quad-Tree

- A natural generalization of the "Beamlet Decorated Quad-Tree" (Donoho and Huo):



- We apply a similar idea in 3D using planelets transform as means of computation of SSEs of sections on different sides of each dividing planelet.
- The Decorated QT in our case is made of dyadic cubes in different scales and their divisions by one significant planelet.

Division Criteria

- The division into two parts of a dyadic cube in certain scale is done by an F-test (assuming white noise), which uses the planelets transform to compute the test statistic:

$$SSE(V)=\sum_{v_i\in V_{voxels}} v_i^2-N_V\cdot\bar{v}^2$$

$$N_V=Volume(V)$$

H0: Good Split
H1: No Split

$$F_{st}(V_1,V_2)=\frac{(SSE(V)-SSE(V_1)-SSE(V_2))/1}{(SSE(V_1)+SSE(V_2))/(N_{V_1}+N_{V_2}-2)}$$

$$F_{st}(V_1,V_2)\sim F_{1,N_V-2}$$

- Computing the p-value for the given test it can easily be decided if division is needed.

The Algorithm

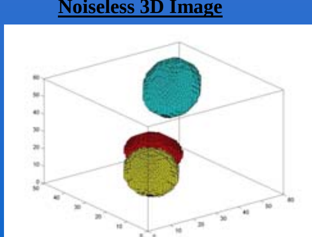
0. Begin with the entire cube, Determine α
1. Find best Planelet Split
2. Check division Criteria (P-value<α)
   a) if holds split and stop
   b) if does test cube homogeneity
      if the cube is homogenous – stop
      else apply a quad split and repeat recursively from 1 for each sub cube.

Remarks

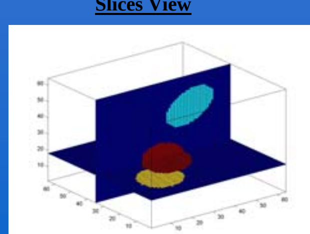
-The algorithm uses a single sensitivity parameter, α, which is statistically meaningful (level of signifficance) -

- The overall complexity of the algorithm is $O(n^3\log(n))$

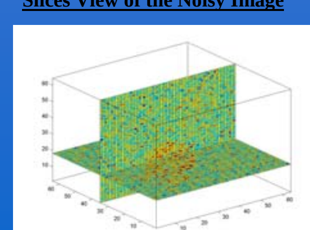
Noiseless 3D Image



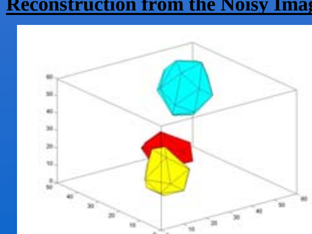
Slices View



Slices View of the Noisy Image



Reconstruction from the Noisy Image



Contact Person: Ofer Levi, levio@bgu.ac.il