

Higher Spin Gauge Theories and Unfolded Dynamics

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Minneapolis, August 01, 2006

Plan

1. Motivation: gauge fields and HS problem
2. Unfolded dynamics
3. Four-dimensional HS models in ten dimensions
4. Conclusions

Relativistic Field Theory

$$\varphi^\nu(x) \in \Gamma(B)$$

B : a bundle over space-time $M^d = G/H$

$M^d = ISO(d)/SO(d)$ **Minkowski**

$M^d = SO(d-1, 2)/SO(d)$ **AdS_d**

$M^d = SO(d, 2)/P$ **compactified Minkowski**

Particles: G -invariant linear equations of motion

$$L\varphi^\alpha(x) = 0.$$

Space of various states of an elementary particle:
space of solutions as an irreducible G -module

Consistency:

- **Unitarity:** QM probabilistic interpretation $p \geq 0$
- **Lowest weight:** Energy boundness

Noncompact G : Quantum fields form infinite dimensional G -modules

$G = SO(d, 2)$: a particle $\sim G$ -module $D(E, \mathbf{s})$:

$\mathbf{s} = s_1 \dots s_{[d/2]}$: $\text{spin}(\mathbf{s}) = \text{weights of } \mathfrak{o}(d)$

E : lowest energy (mass) = weight of $\mathfrak{o}(2)$

Depending on E and \mathbf{s} :

- either a conformal field in M^d
- or a field in AdS_{d+1}

Unitarity: $E \geq E_0(\mathbf{s})$

At $E = E_0(\mathbf{s})$ null states appear that form a G -submodule $D(E', \mathbf{s}')$:

$$D(E_0(\mathbf{s}), \mathbf{s}) = \frac{\lim_{E \rightarrow E_0(\mathbf{s})} D(E, \mathbf{s})}{D(E', \mathbf{s}')}$$

Factorization implies gauge symmetry

The gauge fields are called massless fields ($E = E_0(\mathbf{s})$)

Gauge symmetries

Gauge Symmetries: parameters $\varepsilon^\Omega(x)$

Examples:

- $s = 1, m = 0, E = d - 2$ $A(x) = dx^n A_n(x)$

Maxwell field strength $F(x) = dA(x), \quad d = dx^n \frac{\partial}{\partial x^n}$

Field equations: $d^*F(x) = J(x)$

Gauge transformation: $\delta A(x) = d\varepsilon(x)$

Not all derivatives of fields are fixed by initial data

Lorentz gauge condition: $d^*A(x) = 0$

G-invariant gauge conditions are incomplete

$$0 \longrightarrow \lim_{E \rightarrow E_0} D(E, s) \longrightarrow D(E', s') \longrightarrow 0$$

$D(E', s')$: **leftover gauge transformations** $\square\varepsilon(x) = 0$.

- $s = 2, m = 0, E = d - 1$ $g_{nm} = \eta_{nm} + \kappa h_{nm}$

h_{nm} **is a metric fluctuation** ($\kappa \rightarrow 0$).

Linearized diffeomorphism $\delta h_{nm}(x) = \partial_{(n}\varepsilon_{m)}(x)$

- **Any spin**, $m = 0$, $E = d + s - 3$

$\varphi_{n_1 \dots n_s}(x)$ - **rank** s **double traceless symmetric tensor** $\varphi_n^n m^m k_5 \dots k_s(x) = 0$ (C.Fronsdal (1978))

Gauge transformation:

$$\delta \varphi_{k_1 \dots k_s}(x) = \partial_{(k_1} \varepsilon_{k_2 \dots k_s)}(x), \quad \varepsilon^n_{nk_3 \dots k_{s-1}} = 0$$

$\varepsilon_{k_1 \dots k_{s-1}}$ - **symmetric traceless:** $\delta \varphi_n^n m^m k_5 \dots k_s = 0$

Gauge symmetries with no derivatives on the parameters also play a role:

- **Conformal geometry**

$$g_{nm}(x) \rightarrow \Omega^2(x) g_{nm}(x)$$

Usual (**global**) symmetries with a finite number of parameters are remnants (subsymmetries) of gauge (**local**) symmetries.

Nonlinear Gauge Theories

Free fields (**particles**) described by linear equations do not interact (scatter)

Interactions are described by nonlinear PDE

That the interacting theory should have as many (may be deformed) **gauge symmetries** as its free limit imposes severe restrictions on the sets of fields involved and on the form of nonlinearities

- $s = 1$ **Yang-Mills**: connection 1-forms take values in compact Lie algebras
 - $s = 2$ **Einstein gravity**: one metric tensor
 - $s = 3/2$ **Supergravity**: supermultiplets of fields of spins $0 \leq s \leq 2$ with one spin 2 graviton

HS gauge theories contain spin $s > 2$ gauge fields

Higher Spin Problem

Key questions:

- Any consistent nonlinear HS theory?
- The role and meaning of HS gauge symmetries?

Find a nonlinear theory (i.e., action or field equations) such that

- Free field limit gives correct free field equations for HS fields
- Nonlinear theory has as many gauge symmetries as its free limit
- Diffeomorphism invariant = gravity is included
- Field equations are consistent if overdetermined: no additional constraints on the initial data at the nonlinear level

Higher Spin Gauge Theory

Properties

- **HS** ($s > 2$) appear in infinite sets that all include $s = 0, 2, 4, \dots, \infty$
- **Nonlinear terms contain higher derivatives (up to order s for a spin s field)**
- **A consistent nonlinear HS theory admits AdS_d rather than in Minkowski space-time as a solution (AdS radius shows up in the interaction terms)**
(E.S.Fradkin, M.V. (1987))

Summary of Results: (1990,2003)

HS equations are formulated in terms of

$$W(x|Y) \quad Y_i^A \quad A = 0, 1, \dots, d, \quad i = 1, 2$$

x^n : commutative space-time $n = 0 \dots d - 1$

Y_i^A : noncommutative space

The nontrivial part of the HS field equations describes a noncommutative two-dimensional hyperboloid embedded into the (x, Y) -space. Relativistic fields are moduli of the embedding.

Equivalently, the equations preserve the $sl(2)$ algebra Howe dual to the infinite-dimensional non-Abelian HS algebra.

Gauge invariant field equations that involve infinite sets of fields with higher derivatives are naturally formulated in terms of **Unfolded dynamics approach** which is the main theme of this talk.

Unfolded Dynamics

Unfolded dynamics: multidimensional generalization of first-order form of differential equations

$$\dot{q}^i(t) = \varphi^i(q(t)) \quad \text{initial values: } q^i(t_0)$$

Extension $t \rightarrow x^n$: a prolongation that involves higher differential forms (gauge fields) responsible for gauge symmetries (1988)

$$q^i(t) \rightarrow W^\alpha(x) = \{dx^{n_1} \wedge \dots \wedge dx^{n_p} W_{n_1 \dots n_p}^{\alpha p}(x)\} \quad p = 0, 1, \dots$$

Unfolded equations: first-order OPDE of the form

$$dW^\alpha(x) = G^\alpha(W(x), x), \quad d = dx^n \partial_n$$

$$G^\alpha(W, x) = \sum_{n=1}^{\infty} f^\alpha_{\beta_1 \dots \beta_n}(x) W^{\beta_1} \wedge \dots \wedge W^{\beta_n}$$

For our purpose, the most interesting case is

$$G^\alpha = G^\alpha(W)$$

$d > 1$: **Nontrivial compatibility conditions**

$$G^\beta(W) \wedge \frac{\partial G^\alpha(W)}{\partial W^\beta} = 0$$

equivalent to the generalized Jacobi identities

$$\sum_{n=0}^m (n+1) f^\gamma_{[\beta_1 \dots \beta_{m-n}} f^\alpha_{\gamma \beta_{m-n+1} \dots \beta_m]} = 0$$

Any solution to generalized Jacobi identities: FDA

(Sullivan (1968))

FDA is universal if the generalized Jacobi identity is true independently of a particular value of space-time dimension. The HS FDAs are universal. **Every universal FDA = some L_∞ algebra**

The unfolded equation is invariant under the gauge transformation

$$\delta W^\alpha(x) = d\varepsilon^\alpha(x) + \varepsilon^\beta(x) \frac{\partial G^\alpha(W(x))}{\partial W^\beta},$$

where the gauge parameter $\varepsilon^\alpha(x)$ is a $(p_\alpha - 1)$ -form.

No gauge parameters for 0-forms among W^α

Main properties

- Manifest (HS) gauge invariance
- Invariance under diffeomorphisms
 - Exterior algebra formalism
- Interactions: nonlinear deformation of $G^\alpha(W)$
- Degrees of freedom are in 0-forms at any $x = x_0$
(as $q(t_0)$)
 - infinite-dimensional module dual to the space of single-particle states in QFT
 - natural realization of infinite symmetries with higher derivatives
- Independence of space-time
 - Dynamics is encoded by $G^\alpha(W)$
- Universality

Vacuum

Let ω^α be a set of 1-forms:

$$G^\alpha(\omega) = -f_{\beta\gamma}^\alpha \omega^\beta \wedge \omega^\gamma$$

Consistency: Jacobi identity for a Lie algebra \mathfrak{g}

Unfolded equations: flatness condition

$$d\omega^\alpha + f_{\beta\gamma}^\alpha \omega^\beta \wedge \omega^\gamma = 0$$

The gauge transformation

$$\delta\omega^\alpha(x) = D\varepsilon^\alpha(x) = d\varepsilon^\alpha(x) + f_{\beta\gamma}^\alpha \omega^\beta(x) \varepsilon^\gamma(x)$$

Given flat connection $\omega_0(x)$ is invariant under the gauge transformations with the covariantly constant parameters

$$D_0\varepsilon^\alpha(x) = 0$$

Moduli of the generic solution: $\varepsilon^\alpha(x_0)$ at any given point x_0 : parameters of the finite-dimensional global symmetry \mathfrak{h} of the vacuum connection ω_0 .

Gravity as a gauge theory

To unify gravity with Yang-Mills in a HS gauge theory we use its Cartan formulation:

$$g_{mn} \rightarrow e_n^a \rightarrow e_n^a, \omega_n^{ab}, \quad g_{mn} = e_m^a e_n^b \eta_{ab},$$

η_{ab} , is the constant Minkowski metric ($a, b \dots = 0, 1 \dots d-1$)

$e^a = dx^n e_n^a$ vielbein 1-form

$\omega^{ab} = dx^n \omega_n^{ab}$ Cartan (Lorentz) connection

e_n^a has extra $\frac{d(d-1)}{2}$ components compensated by the $o(d-1, 1)$ local Lorentz symmetry

$$\delta e^a(x) = \varepsilon^{ab}(x) e_b(x) \quad \varepsilon_{ab}(x) = -\varepsilon_{ba}(x)$$

e^a, ω^{ab} : 1-form connection of AdS algebra $o(d-1, 2)$

(MacDowell-Mansouri (1978))

or its flat contraction $iso(d-1, 1)$

$$\omega = e^a P_a + \frac{1}{2} \omega^{ab} M_{ab}$$

$M_{nm} : o(d-1, 1) \subset o(d-1, 2), P_n : o(d-1, 2)/o(d-1, 1)$

$$[P_n, P_m] = \lambda^2 M_{nm}$$

$\lambda^{-1} =$ is the AdS radius. Flat limit: $\lambda \rightarrow 0$

The curvature 2-form is

$$R = d\omega + \omega^2 \equiv R^a P_a + \frac{1}{2} R^{ab} M_{ab},$$

$$R^a = D^{\mathcal{L}} e^a \equiv de^a + \omega^a_b \wedge e^b.$$

$$R^{ab} = \mathcal{R}^{ab} - \lambda^2 e^a \wedge e^b, \quad \mathcal{R}^{ab} = d\omega^{ab} + \omega^a_c \wedge \omega^{cb}$$

ω^{ab} is expressed in terms of (derivatives of) e^a by the zero-torsion condition

$$R^a = 0, \rightarrow \omega = \omega(e, \partial e)$$

$\mathcal{R}_{mn,kl} = e^a_k e^b_l \mathcal{R}_{mn,ab}$ is Riemann tensor (if $R^a = 0$).

The zero-curvature equations describe g -symmetric background (fixed metric) geometry in a coordinate independent way

Minkowski spacetime: $\mathcal{R}_{mn,kl} = 0$

$$AdS_d : \quad R^{ab} = 0, \quad R^a = 0.$$

Free fields unfolded

Let W^α contain p -forms \mathcal{C}^i (e.g. 0-forms) and G^i be linear in ω and \mathcal{C}

$$G^i(\omega, \mathcal{C}) = -\omega^\alpha (T_\alpha)^i_j \wedge \mathcal{C}^j.$$

The compatibility condition implies that $(T_\alpha)^i_j$ form some representation T of g , acting in a carrier space V of \mathcal{C}^i . The unfolded equation is

$$D_\omega \mathcal{C} = 0 \tag{1}$$

$D_\omega \equiv d + \omega$: covariant derivative in the g -module V .

The covariant constancy equation: linear equations in a chosen g -symmetric background described by the flat connection ω : $(D_\omega)^2 = 0$.

g : global symmetry

$$\delta \mathcal{C}^i(x) = \varepsilon^\alpha(x) (T_\alpha)^i_j \mathcal{C}^j(x), \quad D_\omega \varepsilon = 0$$

If \tilde{g} is a larger Lie algebra acting in V , $g \subset \tilde{g}$, it is a symmetry of (1). $\tilde{g} = \text{End } V$ is the maximal symmetry of (1)

Klein-Gordon equation

Minkowski space in Cartesian coordinates: $e_m^a = \delta_m^a$

To unfold spin-0 massless field, introduce infinite set of 0-forms 

$$C_{a_1 \dots a_n} = C_{(a_1 \dots a_n)}, \quad \eta^{bc} C_{bca_3 \dots a_n} = 0.$$

Unfolded KG equation

$$dC_{a_1 \dots a_n} = e^b C_{a_1 \dots a_n b}$$

This system is consistent since $e^b \wedge e^c = -e^c \wedge e^b$

The first two equations

$$\partial_n C = C_n, \quad \partial_n C_m = C_{mn},$$

imply $C_{nm} = \partial_n \partial_m C$. Tracelessness of C_{nm} :

$$\square C(x) = 0.$$

All other equations:

$C_{a_1 \dots a_n} = \partial_{a_1} \dots \partial_{a_n} C$ $C_{a_1 \dots a_n}$: basis of the space of all on-mass-shell nontrivial derivatives of $C(x)$: V is an appropriate quotient of J^∞

$d = 1$: two independent components $q(t) = C(t)$, $p(t) = C_n(t)$ rank $r > 1$ traceless tensors are zero

Spin two: Einstein equations

Fields: 1-forms e^a and ω^{ab} .

Zero-torsion constraint and Einstein equations

$$\star \quad D^{\mathcal{L}} e^a = 0, \quad R^{ab} = e_c \wedge e_d C^{ac, bd}$$

$C^{ac, bd}$ is the Weyl tensor in the symmetric basis

$$C^{(ac, b)d} = 0, \quad \eta_{ac} C^{ac, bd} = 0 \quad \boxplus$$

To unfold the equations for $dC^{ac, bd}$ are needed.

The restrictions on the derivatives of $C^{ac, bd}$ result from the Bianchi identities

$$\begin{aligned} D^{\mathcal{L}} R^{ab} \equiv 0 &\Rightarrow e_c \wedge e_d \wedge (D^{\mathcal{L}} C^{ac, bd}) = 0 \\ &\Rightarrow D^L C^{ac, bd} = e_f C^{acf, bd} \end{aligned}$$

where $D^{\mathcal{L}}$ is Lorentz covariant derivative and

$$C^{(abc, d)f} = 0, \quad \eta_{ab} C^{abc, de} = 0 \quad \boxplus \boxplus$$

The process continues by analysing Bianchi identities to give

$$\star \star \quad D^{\mathcal{L}} C^{a_1 \dots a_{2+k}, b_1 b_2} = \quad \boxplus \boxplus \boxplus \boxplus \boxplus$$

$$e_0 c((2+k)C^{a_1 \dots a_{2+k} c, b_1 b_2} + 2C^{a_1 \dots a_{2+k} (b_1, c b_2)})$$

\star and $\star \star$: the unfolded form of the linearized Einstein equations.

Central On-Mass-Shell Theorem

$$\begin{cases}
 R_1^{a_1 \dots a_{s-1}, b_1 \dots b_t} = & \begin{array}{c} \square \square \square \square \square \square \square \square \\ \square \square \square \square \square \square \square \square \end{array} \begin{array}{l} s-1 \\ t \end{array} \\
 = \delta(t - (s-1)) e_{0c} \wedge e_{0d} C_1^{a_1 \dots a_{s-1} c, b_1 \dots b_{s-1} d} \\
 \widetilde{D}C_1^{a_1 \dots a_{s+k}, b_1 \dots b_s} = 0 & \begin{array}{c} \square \square \square \square \square \square \square \square \square \square \square \square \square \square \\ \square \square \square \square \square \square \square \square \square \square \square \square \square \square \end{array} \begin{array}{l} s+k \\ s \end{array} \\
 \end{cases} \quad \begin{array}{l} 0 \leq t \leq s-1 \\ 0 \leq k \leq \infty \end{array}$$

Infinite set of 0-forms C describe all gauge invariant on-mass-shell nontrivial derivatives.

Nonlinear corrections: nonlinear deformation of the free unfolded system

1-forms and symmetry parameters form the set of $o(d-1, 2)$ -modules

$$\left. \begin{array}{l} \omega_n^{(A_1 \dots A_{s-1}, A_s) B_2 \dots B_{s-1}} = 0 \\ \omega_n^{A_1 \dots A_{s-3} C} C, B_1 \dots B_{s-1} = 0 \end{array} \right\} \begin{array}{c} s-1 \\ \square \square \square \square \square \square \square \square \\ o(d-1, 2) \end{array}$$

This set results from gauging the Eastwood's (2002) algebra of symmetries of $\square C = 0$ in $d-1$ dimensions, which is the simplest HS symmetry algebra of global symmetries: (higher) Killing tensors:

$$D_0 \epsilon^{A_1 \dots A_{s-1}, A_s} B_2 \dots B_{s-1} = 0$$

Dynamical content via σ_- -cohomology

At the free field level unfolded equations are

$$\mathcal{D}_0 C = 0,$$

C being tensors of different ranks.

Let G : be some diagonalizable grading operator in V such that its spectrum is bounded from below: $V = V_0 \oplus V_1 \oplus \dots$. Typically, G counts a rank of a tensor.

Writing

$$(D + \sigma_- + \sigma_+) C(x) = 0, \quad C \subset V$$

$$[G, D] = 0 \quad [G, \sigma_{\pm}] = \pm \sigma_{\pm}$$

σ_- decreases the grading and is algebraic

σ_+ increases the grading

D does not change

$$\mathcal{D}_0^2 = 0 \Rightarrow$$

$$\sigma_-^2 = 0, \quad \sigma_+^2 = 0, \quad D^2 + \{\sigma_-, \sigma_+\} = 0, \quad \{D, \sigma_{\pm}\} = 0$$

Important (conformal) case is with $\sigma_+ = 0$:

D and σ_- form a bicomplex.

Analysis of Bianchi identities = analysis of cohomology $H^p(\sigma_-, V)$ of σ_- acting on $\Lambda M^d \times V$.

Let C be a p -form:

- 1. **Differential gauge symmetry parameters (like diffeomorphisms) :** $H^{p-1}(\sigma_-, V)$
- 2. **Dynamical fields (like metric) in C :** $H^p(\sigma_-, V)$
- 3. **Nontrivial equations (like Einstein equations)**
 $H^{p+1}(\sigma_-, V)$
- 4. **Syzygys :** $H^k(\sigma_-, V) \quad k > p + 1 ?$

Off-shell-system : $\mathcal{H}^{p+1} \equiv 0 : \quad \mathcal{D}_0 C = 0$
only algebraic constraints

On-shell-system : $\mathcal{H}^{p+1} \neq 0 : \quad \mathcal{D}_0 C = 0$
PDE

Comments

- Unfolded version of any linear g -invariant PDE amounts to the covariant constancy equation for sections of the V -bundle on M for some g -module V : classification of g -invariant equations in terms of generically, infinite-dimensional g -modules.

The modules underlying the unfolding of g -invariant systems with finite sets of dynamical fields are induced from various finite-dimensional p_- -modules where $p_- \subset g$ is spanned by elements of non-positive grade in g .

- The decomposition of the covariant derivative $\mathcal{D}_0 C = \mathcal{D} + \sigma_- + \sigma_+$ may (conformal case) or may not (AdS) have a meaning in terms of g i.e. σ_- may not be an element of g . Different choices of the grading change σ_- and can lead to differently looking but equivalent (dual) dynamical systems

Invariant functionals via Q -cohomology

Equivalent form of the compatibility condition

$$Q^2 = 0, \quad Q = G^\alpha(W) \frac{\partial}{\partial W^\alpha}$$

Q -manifolds

Hamiltonian-like form of the unfolded equations

$$dF(W(x)) = Q(F(W(x))) \quad (2)$$

for any function $F(W)$ of W^α .

Invariant functionals

$$S = \int L(W(x)), \quad QL = 0 \quad (2005)$$

Q exactness implies d exactness by (2).

Actions and conserved charges: Q -cohomology

For off-shell and on-shell unfolded systems,
respectively.

4d massless fields

Field variables: (Weyl) 0-forms $C(Y|x)$

$$x^{\alpha\alpha'} \quad Y^A = (y^\alpha, \bar{y}^{\alpha'})$$

$$A = 1, \dots, 4 \quad \alpha = 1, 2; \alpha' = 1, 2$$

Unfolded form of 4d massless field equations

$$(d + \sigma_-)C(Y|x) = 0$$

$$d = dx^{\alpha\alpha'} \frac{\partial}{\partial x^{\alpha\alpha'}} \quad \sigma_- = dx^{\alpha\alpha'} \frac{\partial^2}{\partial y^\alpha \partial \bar{y}^{\alpha'}}$$

dynamical fields via $H^0(\sigma_-) = \text{Ker } \sigma_-$:

$$C(y, 0|x), C(0, \bar{y}|x)$$

Helicity operator $2H = y^\alpha \frac{\partial}{\partial y^\alpha} - \bar{y}^{\alpha'} \frac{\partial}{\partial \bar{y}^{\alpha'}}$

Definite helicity $HC(Y|x) = sC(Y|x)$

Field equations via $H^1(\sigma_-)$:

$$\frac{\partial}{\partial y^{[\alpha}} \frac{\partial}{\partial x^{\beta]\alpha'}} C(y, 0|x) = 0, \quad \frac{\partial}{\partial \bar{y}^{[\alpha'}} \frac{\partial}{\partial x^{\alpha\beta]}} C(0, \bar{y}|x) = 0$$

and $\square C(0, 0|x) = 0$.

Explicit solution

$$C(Y|x) = \exp \left[-x^{\alpha\alpha'} \frac{\partial^2}{\partial y^\alpha \partial \bar{y}^{\alpha'}} \right] C_0(Y)$$

4d symmetries

The Weyl algebra A_4 of differential operators with polynomial coefficients $P(Y, \frac{\partial}{\partial Y})$ acts on the module V of functions of $Y : C(Y)$. $Lie(A_4)$ is the 4d conformal HS symmetry.

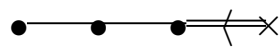
$sp(8|R) \subset Lie(A_4)$:

$$\mathcal{P}_{AB} = \frac{\partial^2}{\partial Y^A \partial Y^B}, \quad \mathcal{L}_B^A = Y^A \frac{\partial}{\partial Y^B} + \frac{\partial}{\partial Y^B} Y^A, \quad \mathcal{K}^{AB} = Y^A Y^B$$

Conformal algebra $su(2, 2) \subset sp(8|R)$ that acts on every spin is the centralizer of the helicity operator H . When all spins are involved as in HS gauge theories, $su(2, 2)$ extends to $sp(8|R)$.

(C.Fronsdal (1985))

The minimal space where $sp(8|R)$ acts geometrically is $Sp(8|R)/P$



$P : (\mathcal{L}, \mathcal{K})$

It is ten-dimensional with local coordinates $X^{AB} = X^{BA}$. To derive manifestly $sp(8)$ invariant 4d field equations in this ten-dimensional space-time use unfolded equations

Four from Ten

$x^{\alpha\alpha'}$ extends to $X^{AB} = (x^{\alpha\alpha'}, x^{\alpha\beta}, x^{\alpha'\beta'})$

10d unfolded equations

$$(dX^{AB} \frac{\partial}{\partial X^{AB}} + \sigma_-) C(Y|X) = 0, \quad \sigma_- = dX^{AB} \frac{\partial^2}{\partial Y^A \partial Y^B}$$

contain the 4d equations in the $dx^{\alpha\alpha'}$ sector. Other equations just determine the dependence on the additional six coordinates.

Only two dynamical fields in $\text{Ker } \sigma_-$:

$C(0|X)$ describes all 4d integer spins

$Y^A C_A(0|X)$ describes all 4d half-integer spins

Nontrivial field equations via $H^1(\sigma_-)$ (2001)

bosons :
$$\left(\frac{\partial^2}{\partial X^{AB} \partial X^{CD}} - \frac{\partial^2}{\partial X^{CB} \partial X^{AD}} \right) C(X) = 0$$

fermions :
$$\left(\frac{\partial}{\partial X^{AB}} C_C(X) - \frac{\partial}{\partial X^{CB}} C_A(X) \right) = 0$$

4d metric appears from the Clifford coordinates among X^{AB} that admit the localization of initial data

Conclusions

- New class of interesting gauge invariant HS models
- The unfolding machinery may have applicability to the analysis of PDEs

Some relevant references

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