

Testing the Poisson Hypothesis and Higher-Order Normal Approximation for Statistics of Poisson-Based Point Process Models

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Abstract. In the first part we study asymptotic Gaussianity of various empirical multiparameter second moment functions (generalizing *Ripley's K-function*) of a homogeneous Poisson process in R^d and show how to use these results for testing the Poisson hypothesis for a given point pattern. In the second part we derive error bounds in the normal approximation of some empirical functionals of independently marked Poisson point processes $\Psi = \sum_{i \geq 1} \delta_{[X_i, M_i]}$ observed in a (large) rectangular window $W_n = \times_{s=1}^d [0, n_s)$, $n = (n_1, \dots, n_d)$, where the i.i.d. marks $(M_i)_{i \geq 1}$ taking values in some mark space are assumed to be strictly bounded. Such functionals are of great relevance in statistical analysis of Poisson cluster processes, dependently thinned Poisson processes (with bounded cluster resp. thinning radius) and Poisson-grain models (with bounded typical grain).

For example, in case Ψ is stationary, we consider unbiased estimators of Palm probabilities and Palm moment measures of these point processes among them the *nearest-neighbour distance function* and *Ripley's K-function*.

The proofs are based on the fact that all of these functionals admit a representation as sum $S_n = \sum_{j \in V_n} f_j(\xi_i, i \in U_j)$ for $V_n = \times_{s=1}^d \{1, \dots, n_s\}$, where the f_j 's are measurable functions of $(m+1)^d$ independent random elements $\xi_i, i \in U_j = \times_{s=1}^d \{j_s, \dots, j_s + m\}$ for some fixed integer $m \geq 1$. For this special class of m -dependent random fields quite a lot of refinements of the central limit theorem (e.g. Berry-Esseen bounds, Edgeworth expansions, large deviations etc.) have been proven in analogy to the situation of mutually independent random variables during the last fifteen years.