

HIGHER ORDER WHITNEY FORMS

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Compatible finite element discretizations of second-order boundary value problems set in the function spaces $H^1(\Omega)$, $H(\mathbf{curl}, \Omega)$, and $H(\text{div}, \Omega)$ will naturally rely on discrete differential forms. Their lowest-order representatives are the well-knowns linear Lagrangian finite elements, edge elements, and face elements. However, given the interior smoothness of the solutions of many boundary value problems, approximation by local polynomials of high degree offers superior efficiency (*hp*-version of FEM).

The first part of the presentation will focus on the algebra behind the construction of higher order Whitney forms. The calculus of differential forms will be used to devise a unified description of discrete differential forms of any order and polynomial degree on simplicial meshes in any spatial dimension. A general formula for suitable degrees of freedom is also available. Fundamental properties of nodal interpolation can be established easily. It turns out that higher order spaces, including variants with locally varying polynomial order, emerge from the usual Whitney-forms by local augmentation.

The second part of the talk will review the concrete construction of suitable basis functions for higher order Whitney forms. Discrete 1-forms will serve as main example. Recent inventions of *p*-hierarchical bases with good conditioning properties will be discussed.

Time permitting, the third part of the talk will investigate *p*-uniform stability properties of high order Whitney forms. It will show how tools from differential geometry and discrete topology can be used to obtain a *p*-uniform discrete Poincaré-Friedrichs inequality that plays a key role in the convergence theory for the *p*-version of discrete differential forms.