UNIQUENESS IN INVERSE PROBLEMS FOR ELASTIC MEDIA

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ABSTRACT. In this talk we consider dynamic inverse problems for bounded, three-dimensional isotropic and anisotropic elastic media with smoothly varying density and elastic properties. Surface data for the inverse problem is modeled by the hyperbolic Dirichlet-to-Neumann map on a finite time interval.

In the case of isotropic elastodynamics we have shown that the compressional and shear wave speeds are determined uniquely by the Dirichlet-to-Neumann boundary data. A new result is the uniqueness of the third parameter for isotropic elastodynamics, the density. Using information about the propagation of "lower-order polarization," we show that the Dirichlet-to-Neumann map determines a certain ray transform. We then apply results of Pestov and Sharafutdinov, who have shown that the ray transform for symmetric tensor fields may be inverted, up to their potential parts, in certain cases.

We next consider elastodynamics for anisotropic and more general elastic media (for example, isotropic elastic media with residual stress). We observe that the transformation of the elastic medium by a change of coordinates, via any diffeomorphism that fixes the boundary, does not preserve the symmetry properties of the elasticity tensor c_{ijkl} and so does not preserve the form of the operator for anisotropic elastodynamics,

$$(Pu)_i \;=\; a(x)\frac{\partial^2 u_i}{\partial t^2} \;-\; \sum_{j,k,l=1}^{3} \partial_{x_j}\left(c_{ijkl}(x)\frac{\partial u_k}{\partial x_l}\right), \tag{1}$$

for example. It follows that change of coordinates is not an obstacle to uniqueness in the case of anisotropic elastodynamics.

We then consider hyperbolic systems of operators of the form (1) with the property that the determinant of the principal symbol can be factored as $\det \sigma_{pr}(P) \;=\; -a(x)^3\; \Pi_k \left(\tau^2 - \xi^t A_k(x)\xi\right)^{r_k}$. We associate the metrics $g_k = (\text{ Sym } A_k)^{-1}$ with these operators; wave propagation occurs along the geodesics of the g_k. We show that, under certain natural conditions, the metrics g_k are determined by the Dirichlet-to-Neumann map in the interior, up to pullback by diffeomorphisms ψ_k that fix the boundary. That is, the geometry of the wave paths is determined, up to isometry, by the Dirichlet-to-Neumann map.

We then apply this result to inverse problems for media modeled by certain hyperbolic systems, and, in particular, to the inverse problem for isotropic elastodynamics with residual stress, modelled here by

$$Pu \;=\; \rho\partial_t^2 u - \nabla_x \cdot \left[\lambda \operatorname{tr}(\nabla_x \otimes u) + \mu(\nabla_x \otimes u) + (\nabla_x \otimes u)^t\big(\mu I + \frac{1}{\rho}\mathcal{T}\big)\right],$$

where $\mathcal{T}(x)$ is a symmetric coefficient matrix. We also show that the nine parameters for elastodynamics with residual stress are determined at the boundary by the Dirichlet-to-Neumann map.