

A Geometric-Optics Proof of a Theorem on Boundary Control Given a Convex Function

Michael Galbraith

July 12, 2001

SCHOOL OF MATHEMATICS
UNIVERSITY OF MINNESOTA
MINNEAPOLIS, MINNESOTA 55455, USA
galbrait@math.umn.edu

Abstract

We consider a hyperbolic equation of the form

$$u_{tt} - \sum_{i,j=1}^n \frac{\partial}{\partial x_i} \left(a_{ij}(x) \frac{\partial u}{\partial x_j} \right) + \text{lower order terms} = 0$$

in a cylindrical region $\Omega \times (0, T)$ in space-time. In the important paper “Inverse/Observability Estimates for Second-Order Hyperbolic Equations with Variable Coefficients” by Lasiecka, Triggiani and Yao [5], those authors use Carleman estimates to show that the equation can be controlled from a subset of the boundary $\partial\Omega \times (0, T)$ if there is a positive function v on Ω which is strictly convex with respect to the metric defined by the coefficients of the equation, if that convex function has non-positive outward normal derivative on the uncontrolled part of the boundary. The time needed for control is a function of the maximum value of v on Ω and a lower bound on its convexity. In the present paper we will show that control in the same time is established by a simpler geometric optics argument—in fact it comes down to a short calculus computation on the value of v along a bicharacteristic of the equation.

References

- [1] Bardos, Claude, Gilles Lebeau and Jeffrey Rauch: Sharp sufficient conditions for the observation, control, and stabilization of waves from the boundary. *SIAM J. Control Optim.* **30** (1992), 1024–1065.
- [2] Duff, G. F. D.: *Partial Differential Equations*. University of Toronto Press, 1956.

- [3] Gulliver, Robert, and Walter Littman: Chord uniqueness and controllability: The view from the boundary, I. *Differential Geometric Methods in the Control of Partial Differential Equations* (Boulder, CO, 1999), 145–175, *Contemporary Mathematics* **268**, AMS, Providence, 2000.
- [4] Hörmander, Lars: *The Analysis of Linear Partial Differential Operators, vol. III* Springer-Verlag, Berlin 1985.
- [5] Lasiecka, Irene, Roberto Triggiani and Peng-Fei Yao: Inverse/observability estimates for second-order hyperbolic equations with variable coefficients. *J. Math. Anal. Applications* **235** (1999), 13-57.
- [6] Lebeau, Gilles: Contrôle analytique. I. Estimations à priori. *Duke Math. J.* **68** (1992), 1–30.
- [7] Lebeau, Gilles and Luc Robbiano: Stabilisation de l'équation des ondes par le bord. (French), *Duke Math. J.* **86** (1997), no. 3, 465-491.
- [8] Littman, Walter: Near-optimal-time boundary controllability for a class of hyperbolic equations. *Control Problems for Systems Described by Partial Differential Equations and Applications* (Gainesville, Fla., 1986), 307–312, *Lecture Notes in Control and Inform. Sci.* **97**, Springer, Berlin-New York, 1987.
- [9] Ralston, James: Gaussian beams and the propagation of singularities. *Studies in Partial Differential Equations* (W. Littman, ed.), *MAA Studies in Mathematics* **23** (1982), 206–248.