Computational analysis for organ bio-fabrication in tissue engineering

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Motivation and Background

Road map for organ printing

Tissue fusion of vascular tissue spheroids

Mathematical models

We use a phase variable to label each fluid,

\[ \phi = \begin{cases} 
1, & \text{in fluid 1 (Cell-aggregates)}, \\
0, & \text{in fluid 2 (Ambient fluid)}. 
\end{cases} \]

We denote the average velocity by \( \mathbf{v} \).
The transport equation for the mass and momentum of the mixture system is given by

\[ \nabla \cdot \mathbf{v} = 0, \]

\[ \frac{D \mathbf{v}}{Dt} = \nabla \cdot \left( \mu \nabla \mathbf{v} + \mathbf{f} \right), \]

where

\[ f = \frac{1}{2} \kappa \nabla \phi^2 + \kappa_{0} \phi (1 - \phi)^2 \]

\[ \tau_1 = 2 \eta_1 \mathbf{D}, \quad \tau_2 = 2 \eta_2 \mathbf{D}, \quad \mathbf{D} = \frac{1}{2} ( \nabla \mathbf{v} + \nabla \mathbf{v}^T ) \]

where \( \eta_1, \eta_2 \) are the viscosity for fluid 1 and 2, respectively.

Boundary conditions:

\[ \nabla \phi \cdot \mathbf{n} |_{\Omega_1} = 0, \quad [\mathbf{v} - \lambda \nabla \phi^{\|}] \cdot \mathbf{n} |_{\Omega_2} = 0, \quad \mathbf{v} |_{\partial \Omega} = 0. \]

Numerical Simulations

Front tracking simulations of the fusion of two tissue spheroids. The two components are the cellular material and the surrounding tissue culture medium.

Morphogenesis of a branching vascular construct made of layer-by-layer deposition in a designed Y-shape pattern.

Morphogenesis of a tubular construct made from the layer-layer deposition of bio-inks (spheroidal cell-aggregates). The computation is done on a 129x129x129 grid using a Galerkin spectral method.

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