Analysis of a Cartesian PML approximation to an acoustic scattering problem.

We consider a Cartesian PML approximation to solutions of acoustic scattering problems on an unbounded domain in $\mathbb{R}^2$ and $\mathbb{R}^3$. The perfectly matched layer (PML) technique modifies the equations outside of a bounded domain containing the region of interest. This is done in such a way that the new problem (still on an unbounded domain) has a solution which agrees with the solution of the original problem. The new problem has a solution which decays much faster, thus suggesting replacing it by a problem on a bounded domain. The perfectly matched layer (PML) technique, in a curvilinear coordinate system and in Cartesian coordinates, has been studied for acoustic scattering applications both in theory and computation. Using a different approach we extend the results of Kim and Pasciak concerning the PML technique in Cartesian coordinates. The exponential convergence of approximate solutions as a function of domain size and/or the PML "strength" parameter, $\sigma_0$, is also shown. We note that once the stability and convergence of the (continuous) truncated problem has been established, the analysis of the resulting finite element approximations is then classical. Finally, the results of numerical computations illustrating the theory, in terms of efficiency and parameter dependence of the Cartesian PML approach will be given.