DYNAMICS AND STABILITY OF LOW REYNOLDS NUMBER SWIMMING NEAR A WALL

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Abstract

The motion of microorganisms and of micro-scale robotic swimmers is governed by low Reynolds number (Re) hydrodynamics, where viscous effects dominate and inertial effects are negligible. The theory of low-Re locomotion has been studied extensively for unbounded fluid domain whereas in reality, microswimmers often move in confined environments and their dynamic behavior is significantly affected by the boundary. In this study, we investigate the dynamics of a simple model of low-Re swimmer propelled by rotating spheres near a plane wall. We highlight the relation between reversing symmetry of the swimmer and the dynamic stability of translation parallel to the wall. The results are demonstrated experimentally on a micro-scale robotic swimmer moving in a highly viscous fluid.

Problem formulation

Locomotion in fluid = internal motion + interaction with fluid

Reynolds number: Re = \frac{\rho U L}{\mu}

- \frac{U}{L} - characteristic velocity \times length, \nu - kinematic viscosity of fluid

Micro-scale swimmer in water: Re \text{Re} \rightarrow 10^{-3} ... \text{Low-Re hydrodynamics}

Swimmer model: assemblage of spheres attached to a thin frame (body).

Input: actuated rotation of spheres about their attachment point to body.

Asymmetric planar motion: Coordiates \( q(t) = (x(t), y(t)) \), plane wall at \( y = 0 \)

Fluid motion - Stokes equation (Re=0): \( \frac{\partial v}{\partial t} + (v \cdot \nabla) v = -\nabla P + \frac{1}{\rho} \nabla \times (\nabla \times v) \)

Boundary conditions (no-slip): \( \frac{\partial v}{\partial r}(r = 0) = 0 \), \( v \times \mathbf{n} = 0 \) on wall

Hydrodynamic force and torque on \( n \) sphere:

\[ F_n = r_n \cdot D_n \]

\[ T_n = \int (r_n \times D_n) \cdot dS \]

where \( r_n = r \), \( D_n = cr \), \( c = \frac{1}{3} \frac{2}{\text{Re}} r \sin^2(\theta) + \frac{1}{3} \frac{2}{\text{Re}} r \sin^3(\theta) \sin(\theta) \)

Quasi-steady motion: each sphere is in static equilibrium (no inertia)

Constraints: linear relation of forces/torques acting on solid bodies to their angular velocities (Brennen, 1982)

Formulation of swimming

Kinematics: particle's velocity = body velocity + internal (actuated) velocity

\[ \mathbf{V} = \mathbf{V}_b + \mathbf{V}_i \]

where \( \mathbf{V}_b \) = body velocity, \( \mathbf{V}_i \) = internal velocities, input

Zero net force and torque on swimmer's body:

\[ \sum F = 0 \]

The swimming equation:

\[ \frac{d}{dt} \mathbf{V}_b + \frac{d}{dt} \mathbf{V}_i = \mathbf{F}_b + \mathbf{F}_i \]

A driftless nonlinear control system


The 2+1 sphere swimmer

When \( \omega_i \neq \omega_j \neq \omega_k \) (equal and opposite angular velocities): in unbounded fluid – swims along axis of symmetry \( z \)

Near a wall – rotates and deviates away from wall

Is pure translation parallel to wall possible?

Is it dynamically stable under perturbations?

Geometric observations:

1. Dynamics is independent of \( \mathbf{U} = (U_1, U_2, U_3) \)

2. Reversing symmetry under reflection about \( x-z \) plane

\[ \rho \mathbf{r} = \rho \mathbf{r} \]

Implication: \( \mathbf{U} = \mathbf{U} \)

For \( y \neq y_i \) \( \mathbf{U} \) is a relative equilibrium point (i.e. \( u \neq u \))

Reversing-symmetric linearized dynamics under \( u = u \)

For sufficiently large \( \eta_p \), \( \lambda = 0 \) \( \Rightarrow \) two imaginary eigenvalues \( \lambda = \pm |\lambda| \text{e}^{-\tau} \text{e}^{\pm i\theta} \)

Trajectories in \( y-z \) plane under different initial conditions and \( u = u \)

The 2+1 cylinder swimmer – breaking fore-aft symmetry:

Break for-all symmetry by adding unactuated sphere in front

Steady parallel motion at a shifted orientation \( \theta \neq 0 \)

Linearization eigenvalues shift to LHP

Implications: stable micro-swimming without on-board sensing and control explain microorganisms’ attraction to surfaces (Borha et al., 2008)

Summary

Swimmer model with rotating spheres – constant shape, planar motion

Study existence and stability of steady translation parallel to wall

Swimmer fore-aft symmetry – marginal stability, periodic oscillations

Breaking fore-aft symmetry = passive asymptotic stability

Experiments: micro-scale model, rotating cylinders, viscous fluid

Results agree qualitatively with theoretical predictions

In future – extend to shape-changing swimmers near wall

Acknowledgments:

Fulbright and Bikura postdoctoral scholarships (Y. Or)

Support of Boeing Corporation (R. Murray)

Sebastian Zhang, ME senior thesis project at Caltech

Experimental Results

Macro-scale robotic swimmer in silicone oil (viscosity \( \nu = 0.040 \text{kg m}^{-1} \text{s}^{-1} \))

Rotating two acrylic cylinders mounted on floating frames to ensure 2D motion

DC Motors + gear reduction, rotate at 0.5Hz in oil for 6V input

Reynolds number: \( Re = \frac{U L}{\nu} \), \( \nu = 0.005 \)

Motion tracked by IR camera + reflective markers on the robot

Swim in middle of tank with speed 1.5mm/s at equal input voltages of 6V

The two-cylinder swimmer – for-all symmetric:

Robotic swimmer prototype

Steady parallel swimming for equal inputs \( 6 \text{V} \)

Periodic oscillatory motion

For unequal inputs \( 6 \text{V} \)

Vortex (spiraling) motion

Qualitative agreement of experimental results and theoretical predictions

Limitations in current experimental system (to be improved):

No direct control of input angular velocities

Flatting of wires changes ratio of voltage to \( \omega \) \( \Rightarrow \) trajectories intersect

Observe off-plane til motion due to frame-wall interaction

Limited swimming length (plan to change to a circular container)


Movies available at www.technion.ac.il/~izi/research/low_Re_swim/acc2010

The two-sphere swimmer

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