Effects of variable viscosity and viscous dissipation on the disappearance of criticality of a reactive third-grade fluid in a slab

Samuel S. Okoya
Department of Mathematics
Obafemi Awolowo University, Ile - Ife, 220005, NIGERIA
E - mail: sokoya@oauife.edu.ng, ssokoya2003@yahoo.it
Tel. No. +234 803 471 9511

1. The Equations

The one-dimensional equations for reactive fully developed flow of an incompressible, thermodynamically compatible fluid of grade three are:

$$\frac{d}{dy}\left(\exp(-\gamma\theta)\frac{du}{dy}\right) + 6\Lambda\left(\frac{du}{dy}\right)^2\frac{d^2u}{dy^2} = C, \tag{1}$$

$$\frac{d^2\theta}{dy^2} + \Gamma\left(\frac{du}{dy}\right)^2\left(\exp(-\gamma\theta) + 2\Lambda\left(\frac{du}{dy}\right)^2\right) + \delta(1+\beta\theta)^m\exp\left(\frac{\theta}{1+\beta\theta}\right) = 0, \tag{2}$$

together with the corresponding boundary conditions

$$u(-1) = 0, \quad u(1) = 0, \quad \textbf{and} \quad \theta(-1) = 0, \quad \theta(1) = 0, \tag{3}$$

*** We assume heat generation with variable pre-exponential factor (see Massoudi and Phuoc (2008)).**

*** The viscosity is temperature dependent with $\gamma \geq 0$ for a liquid and γ**

< 0 for a gas.

Observation: It is worth noting that the third term in equation (2) represents the heat generation term that has been excluded from Szeri and Rajagopal (1985). In addition, if $\gamma = 0$, we are in Okoya 2006.

2. Computational method and confirmations

The velocity equation (1) and the temperature equation (2) are coupled and must be solved simultaneously. By means of the shooting method, to some extent modified to cope with given type of non-linearities, the dimensionless governing equations (1) - (2) and the boundary conditions (3) will be numerically investigated with the unknown parameter taken to be δ. We are interested in studying the bifurcation of the physical parameters (δ, β, θ_{max}), where θ_{max} is the value of the maximum temperature of the central plane. It is a well known feature that criticality is represented by discontinuous response of $\theta_{max}(\delta)$ when $\beta \neq 0$ (but $\beta < \beta_{tr}$), while criticality disappears for $\beta = \beta_{tr}$, with smooth responses of θ_{max} to δ for higher values of β. A test of the accuracy of the numerical procedure is provided by comparing the results against those for special cases in the literature, i.e. Boddington et al. (1983) and (1984), Shonhiwa and Zaturska (1987) and Okoya (2006).

3. Results

The computed results shown on Figures 1, 2, 3, 4, 5 and 6 (extracted from a forthcoming paper) represent the variation of transitional parameters with respect to the non-Newtonian parameter Λ, viscous heating Γ and the viscosity variational number γ

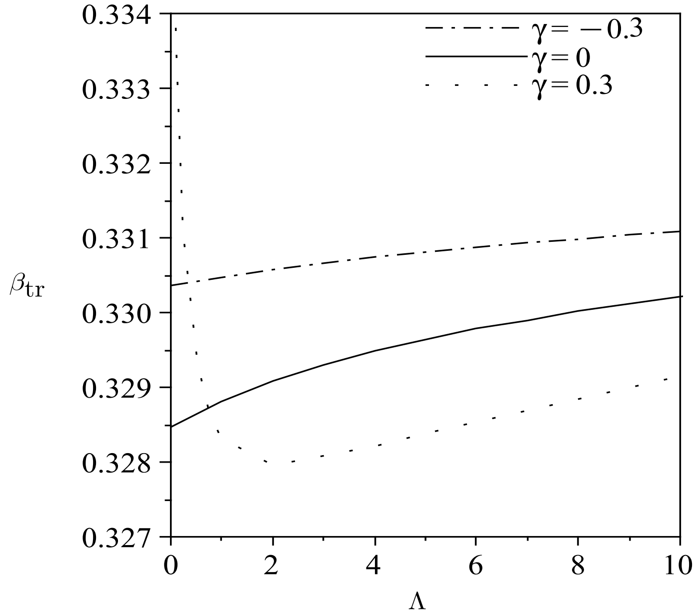

Figure 1: Illustrating the profiles of β_{tr} with Λ for C = -0.25, m = 0.5, Γ = 16.

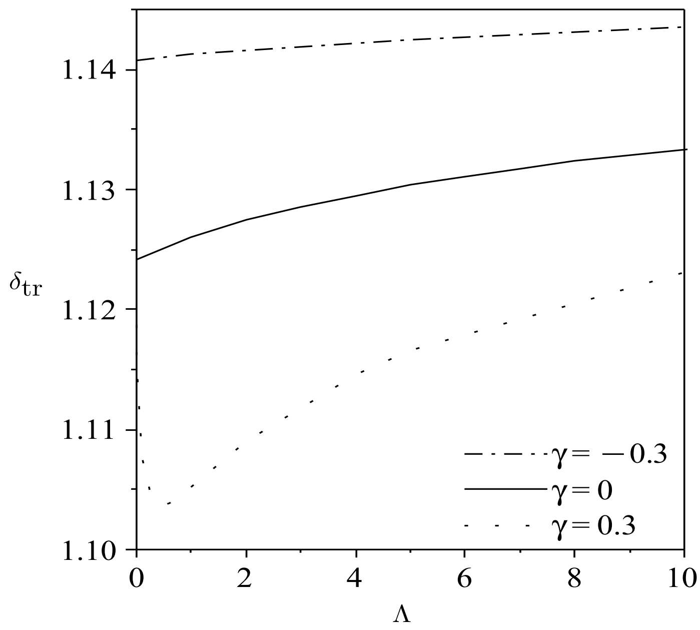

Figure 2: Illustrating the plots of δ_{tr} versus Λ for C = -0.25, m = 0.5, Γ = 16.

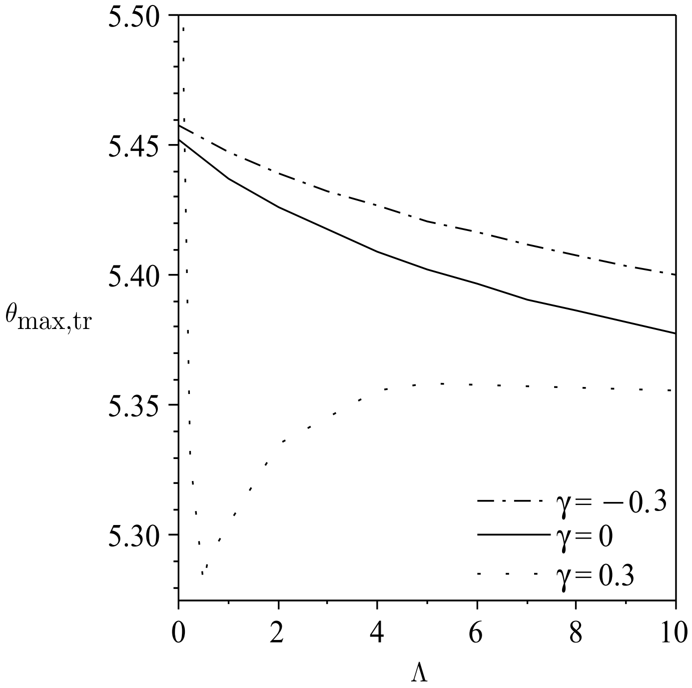

Figure 3: Illustrating the dependence $\theta_{max,tr}$ on Λ for C = -0.25, m = 0.5, Γ = 16.

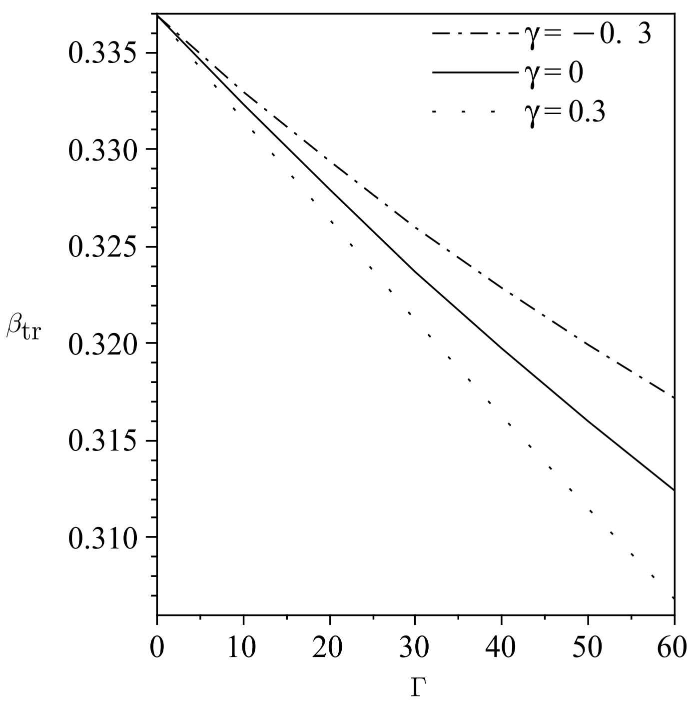

Figure 4: Illustrating the comparisons of β_{tr} with Γ for C = -0.25, m = 0.5 and Λ = 5.

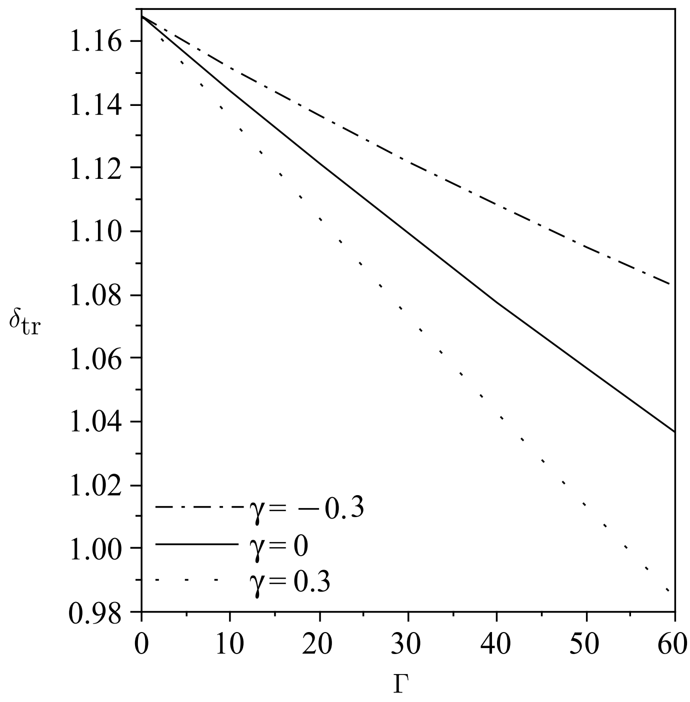

Figure 5: Illustrating the plots of δ_{tr} versus Γ for C = -0.25, m = 0.5 and Λ = 5.

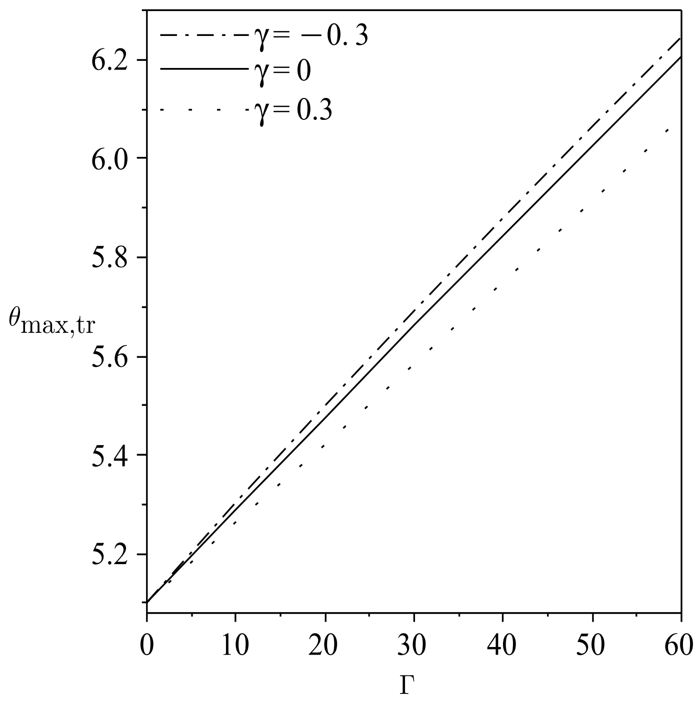

Figure 6: Illustrating the pictures of $\theta_{max,tr}$ on Γ for C = -0.25, m = 0.5 and Λ = 5.

References

[1] T. Boddington, C. G. Feng and P. Gray, Thermal explosions, criticality and the disappearance of criticality in systems with distributed temperatures I. Arbitrary Biot number and general reaction - rate laws, Proc. R. Soc. Lond., A 390 (1983) 247 - 264.

[2] T. Boddington, C. G. Feng and P. Gray, Thermal explosions, criticality and the disappearance of criticality in systems with distributed temperatures II. An asymptotic analysis of criticality at the extremes of Biot number ($(Bi \to 0, (Bi) \to \infty)$ for general reaction rate - law, Proc. R. Soc. Lond., A 392 (1984) 301 - 322.

[3] M. Massoudi and T. X. Phuoc, Flow of a non - linear (density - gradient - dependent) viscous fluid with heat generation, viscous dissipation and radiation, Math. Meth. Appl. Sci., 31 (14) (2008) 1685-1703.

[4] S. S. Okoya, Thermal stability for a reactive viscous flow in a slab, Mechanics Research Communications, 33 (5) (2006) 728 - 733.

[5] T. Shonhiwa and M. B. Zaturska, Disappearance of criticality in thermal explosion for reactive viscous flows, Combustion and Flame, 67 (1987) 175 - 177.

[6] A. Z. Szeri and K. R. Rajagopal, Flow of a non - Newtonian fluid between heated parallel plates, Int. J. Non - Linear Mechanics, 20 (2) (1985) 91 - 101.