The Navier-Stokes-\(\alpha\) model of turbulence is a mollification of the Navier-Stokes equations in which the vorticity is advected and stretched by a smoothed velocity field. The smoothing is performed by filtering the velocity field over spatial scales of size smaller than \(\alpha\). The statistical properties of the smoothed velocity field are expected to match those of Navier-Stokes turbulence for scales larger than \(\alpha\).

For wavenumbers \(k\) such that \(k\alpha \gg 1\), corresponding to spatial scales smaller than \(\alpha\), there are three candidate power laws for the energy spectrum, corresponding to three possible characteristic time scales in the model equations. The three possibilities depend on whether the time scale of an eddy of size \(k^{-1}\) is determined by \((k|u_k|)^{-1}\), \((k|v_k|)^{-1}\), or \((k\sqrt{(u_k, v_k)})^{-1}\), where \(u_k\) and \(v_k\) are the components of the filtered velocity field \(u\) and unfiltered velocity field \(v\), respectively, for wavenumber \(k\). Determining the actual scaling requires resolved numerical simulations.

We measure the scaling of the energy spectra from high-resolution simulations of the two-dimensional Navier-Stokes-\(\alpha\) model, in the limit as \(\alpha \to \infty\). The energy spectrum of the smoothed velocity field scales as \(k^{-7}\) in the direct enstrophy cascade regime, consistent with the dynamics dominated by the time scale given by \((k|v_k|)^{-1}\). We are thus able to deduce that the dynamics of the dominant cascading conserved quantity, namely the enstrophy of the rough velocity, determines the power law for small scales.

For the two-dimensional Leray-\(\alpha\) model, the time scale given by \((k\sqrt{(u_k, v_k)})^{-1}\) is understood to characterize the dynamics of the conserved enstrophy. Indeed, our numerical simulation of this model gives a \(k^{-5}\) power law in the enstrophy inertial subrange. This result supports our claim regarding the characteristic time scale of the two-dimensional NS-\(\alpha\) model for wavenumbers \(k\alpha \gg 1\).