We develop a theory on DNA electrophoresis that shows stretch-dependent electrophoretic mobility in agreement with an experiment observation. In our theory, a DNA molecule is modeled as a freely-jointed chain, each of whose segments consists of a collinear series of charged spheres, which we call a "shish-kebab" segment. First, by calculating the interaction between charged spheres in an electric field, we show that the electrophoretic mobility of a shish-kebab segment is dependent on the molecular conformation (Fig. 3). The result shows an enhancement of the magnitude of the electrophoretic mobility under the stretch of the DNA molecule.

**Model for a flexible molecule**

In order to estimate the electrophoretic velocity of a charged polymer, we connect together shish-kebabs, each of length equal to a single Kuhn step length of the polymer molecule, to assemble a flexible polymer, in the same way as rods are connected together to form a freely-jointed-chain.

**Discussion**

Experimental results show that in an electric field gradient, DNA molecules are not always uniformly stretched; some are coiled, dumbbell-shaped, half-dumbbell-shaped, or folded. This diversity of DNA conformations results in variability of the electrophoretic mobility, since the orientation distribution of the DNA segments relative to the electric field affects the magnitude of the enhancement in the electrophoretic mobility according to our model. However, since a molecule tends to stretch in the direction of the electric field because of the gradient of the field magnitude, the electrophoretic mobility is always enhanced (or amplified) with stretching regardless of the particular DNA conformation.

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**Figure 1.** The electrophoretic velocity of an insulating charged particle of arbitrary shape in an uniform electric field $E$ is given by $v_{EP}=\frac{\varepsilon_0 E \zeta}{\eta}$, and independent of the shape and the size of the particle.

**Figure 2.** (a) The measurement of instantaneous electrophoretic mobility, where $E$ is the magnitude of the local electric field; (b) dimensions of a converging microchannel used in experiments.

**Figure 3.** Instantaneous electrophoretic mobility data for 50 A-DNA molecules (a) as a function of DNA length and (b) as a function of DNA configuration and its configuration dynamics.

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**Equations**

- $v_{EP}=\frac{\varepsilon_0 E \zeta}{\eta}$
- $\mu_{eff}=\frac{v_{EP}}{E}$
- $\zeta=\frac{Q}{e}$
- $E=\frac{F}{q}$

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**References**


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**Results**

If we take an ensemble average over the angles $\theta$ of each shish-kebab, the center-of-mass migration velocity of a single shish-kebab can be written as:

$$v_{CM}(\theta) = \frac{Q}{2 \pi} \cos \theta$$

where $Q$ is the charge and $\theta$ is the angle between the particle surface and the direction of the electric field.