Spherical Bubble Collapse in Viscoelastic Fluids

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1. Introduction
An understanding of the dynamics of cavitation bubbles in viscoelastic fluid is crucial to many areas of science and engineering.

Application areas of interest include:
- Cavitation damage in journal bearings
- Acoustic depolymerisation
- Lithotrispy
- Angioplasty

The study of spherical dynamics allows one to gain important insights into the effects of viscoelasticity on cavitation bubbles.

In most instances in nature, bubble dynamics are non-spherical. Here we present a numerical method that describes the essential features of spherical viscoelastic dynamics, that can be extended to non-spherical cases.

2. Governing Equations
Assuming incompressibility and irrotationality, the governing equations are:

Conservation of mass
\[ \nabla \cdot \mathbf{p} = 0 \]

Equation of Motion
\[ \frac{D\mathbf{u}}{Dt} = \frac{1}{2} \left| \nabla \left( \mathbf{u} - \mathbf{T} \right) \right|^2 - T_{nn} + p_c - p_b \]

Constitutive Equation
\[ \frac{D\mathbf{T}}{Dt} + T_{nn} = \frac{1}{Re} \mathbf{\gamma}_{nn} \]

Here, \( \mathbf{u} \) is the velocity potential, \( \mathbf{T} \) the normal component of the extra stress, \( \mathbf{T}_{nn} \) the normal component of the rate of deformation, and \( p_c, p_b \) the pressures in the bubble and at infinity respectively.

\( Re \) is the Reynolds number, a measure of the fluid’s viscosity. 
\( De \) is the Deborah number, a measure of the fluid’s elasticity.

Non-spherical bubble dynamics can be described, but we present only the spherical case here.

3. Newtonian Fluid
The variation of bubble radius with time for a Newtonian fluid is shown in Fig. 1. The increase in viscosity (decrease in Reynolds number \( Re \)) results in an increase in the collapse time.

![Fig. 1. Newtonian bubble collapse](image1)

4. Maxwell Fluid
Collapse in viscoelastic fluids is characterised by the damped oscillation of the bubble radius with time (see Fig. 2).

Larger Deborah numbers produce larger amplitudes of oscillation.

![Fig. 2. Viscoelastic bubble collapse](image2)

5. General Viscoelastic Fluids

More general models are amenable to solution by this technique.

Fig. 5 shows the oscillation of bubble radius in a Jeffreys type fluid. (A Maxwell fluid with a Newtonian solvent contribution). Increasing the solvent viscosity decreases the amplitude of oscillation. The frequency of oscillation remains approximately constant.

The dynamics of the Doi-Edwards and Rouse models are shown in Fig. 6. These are more general models that better describe polymer solutions and melts. In both cases elastic forces are not large enough to produce large oscillations.

![Fig. 3. Near-elastic bubble oscillation](image3)

![Fig. 4. The different bubble dynamic about the rebound condition Eqn. (1).](image4)

For large Deborah numbers there exists an elastic rebound condition. For rebound to occur we require

\[ De Re < \frac{4}{3} \] (1)

Fig. 3 shows the near elastic oscillation of the bubble radius with time when Eqn. 1 is satisfied. Fig. 4 shows the different bubble dynamic above and below the rebound limit.

6. Summary
The behaviour of bubbles in viscoelastic fluid is markedly different from the Newtonian case. The dynamics are characterised by the damped oscillation of bubble radius with time. The numerical method used describes the essential features of spherical dynamics and can be readily extended to non-spherical geometries to investigate the collapse of bubbles near solid boundaries, for example.