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**Diffusion PDEs for Image Processing**

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**Perona-Malik:** Identify edges using $|\nabla u|$, and inhibit diffusion near edges.

$$u_t = \nabla \cdot (g(|\nabla u|) \nabla u), \quad g(s) = \frac{1}{1 + c^2 s^2}, \quad c > 0$$
Fourth Order Denoising Models

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My work combines a novel regularization $|\nabla^{1-\epsilon} u|$ with a fourth order model to create a denoising PDE which is well-posed and avoids the artifacts associated with Perona-Malik, as well as the splotchiness observed with other fourth order models.