Dewetting of Thin Liquid Films

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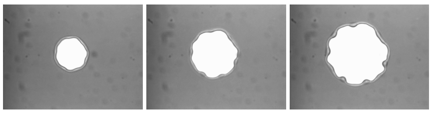
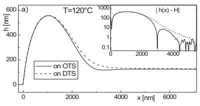
Rim Morphology in Dewetting Experiments

Rim instability.



Experiments Karin Jacobs & Chiara Neto – Soft Condensed Matter, University of Saarland: Dewetting of PS from OTS covered Si.

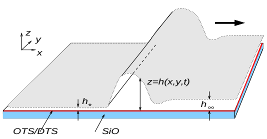
Changes in the rim profile.



(Karin Jacobs, Renate Fetzer) Ridge profiles of 130 nm PS films on DTS and OTS covered Si wafers at constant temperature.

- Changing OTS to DTS changes tail of profile.
- OTS-DTS: Different brush length
- Different dewetting speed suggests different effective slip at liquid-solid interface.

Intermolecular driving forces



$$\phi(h)=\frac{c_s}{h^8}-\frac{A}{h^2}$$

Thin Film Models

$$\begin{aligned}-\partial_x p+\varepsilon_\ell^2\partial_{xx}u+\partial_{zz}u&=0,\\ -\varepsilon_\ell^{-1}\partial_z p+\varepsilon_\ell^3\partial_{xx}w+\varepsilon_\ell\partial_{zz}w&=0,\\ \partial_x u+\partial_z w&=0.\end{aligned}\qquad 0<z<h(x,t)$$

$$\begin{aligned}\partial_z u+\dots&=0,\\ p-\phi'(h)+\frac{\partial_{xx}h}{\left(1+\varepsilon_\ell^2(\partial_x h)^2\right)^{3/2}}+\dots&=0\\ \partial_t h-w+u\partial_x h&=0,\end{aligned}\qquad z=h(x,t)$$

$$u=b\,\partial_z u\quad\text{and}\quad w=0.\qquad z=0$$

Scale Separation: $\varepsilon_\ell=\frac{\mathbb{H}}{\mathbb{L}}\ll 1$; Balances: $\varepsilon_\ell\frac{\mathbb{P}\mathbb{H}}{\mu\mathbb{U}}=1$, $\varepsilon_\ell^2\frac{\sigma}{\mathbb{H}\mathbb{P}}=1$.

Weak-slip lubrication model

$$h_t+\partial_x\left(m(h)\left(h_{xxx}-\partial_x\phi'(h)\right)\right)=0,\quad \text{mobility } m(h)=h^3+bh^2.$$

$b=0:\quad m(h)=h^3$, **No-slip model**

$b\to\infty:\quad m(h)=h^2$, **Intermediate slip model**.

Thin Film Models

New balance: $\frac{\mathbb{P}\mathbb{H}}{\mu\mathbb{U}}=\varepsilon_\ell$. Assume: $b=\beta/\varepsilon_\ell^2=O(\varepsilon_\ell^{-2})$.

$$\begin{aligned}-\varepsilon_\ell^2\partial_x p+\varepsilon_\ell^2\partial_{xx}u+\partial_{zz}u&=0,\\ -\partial_z p+\varepsilon_\ell^2\partial_{xx}w+\partial_{zz}w&=0,\\ \partial_x u+\partial_z w&=0.\end{aligned}\qquad 0<z<h(x,t)$$

$$\dots\qquad z=h(x,t)$$

$$\varepsilon_\ell^2 u=\beta\,\partial_z u\quad\text{and}\quad w=0.\qquad z=0$$

Strong-slip model

$$h_t+(uh)_x=0,\qquad -\frac{4}{h}\partial_x(hu_x)+\left(h_{xxx}-\partial_x\phi'(h)\right)-\frac{u}{\beta h}=0$$

Münch/B. Wagner/Witelski [J. Eng. Math., 2005]

- $\beta\to 0$: recover intermediate slip model.
- $\beta\to\infty$: recover suspended film model.

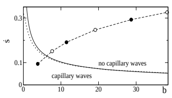
  Erneux/Davis [Phys. Fluids 1993]; Howell [PhD-Thesis 1994]; Ida/Miksis [Appl. Math. Letters 1996]; Brenner/Gueyffier [Phys. Fluids 1999]; $\cdots$

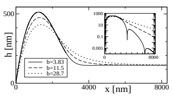
Rim Profile – Strong Slip

Characteristic equation:

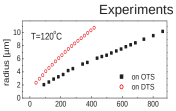
$$\lambda^3+4\dot s\lambda^2-\dot s/\beta=0.$$

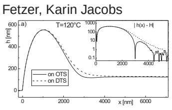
There are two modes with Re$(\lambda)<0$; complex conjugate, iff $\dot s^2<\frac{27}{256}\frac{1}{\beta}$.





Experiments Renate Fetzer, Karin Jacobs

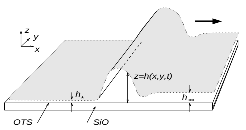




- Fetzer/Jacobs/Münch/Wagner/Witelski [PRL 2005].
- Quantitative method: Fetzer/Münch/Wagner/Rauscher/Jacobs. [Langmuir 2007].

Slippage and the Morphology of the Dewetting Ridge

Contact-line instability



$$h_t+\nabla\cdot\left[h^n\nabla\left(\Delta h-\phi'(h)\right)\right]=0,$$

$$\lim_{x\to-\infty}h(x,y,t)=h_*=\varepsilon h_\infty,$$

$$\lim_{x\to+\infty}h(x,y,t)=h_\infty.$$

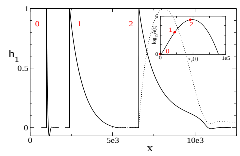
Intermediate slip: n=2

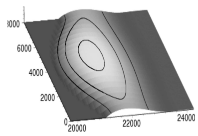
(No Slip: n=3)

Linear Stability Analysis

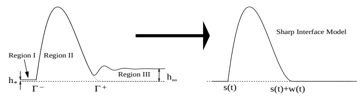
Ansatz: $h(x,y,t)=h_b(x,t)+\delta h_1(x,t)\exp(iky),\qquad \delta\ll 1.$

- h_b: time dependent base state; k wavenumber $(=2\pi/l)$.
- $O(\delta)$: $\partial_t h_1=\mathcal{L}_x(h_b(x,t),k)h_1$
- Monitor shape of $h_1(x,t)$ and $A(t):=\frac{\max_x|h_1(x,t)|}{\max_x|h_1(x,t_0)|}$





Sharp Interface Model

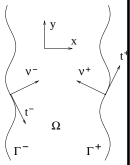


$$h_t=-\nabla\cdot(h^2\nabla\triangle h),\quad\text{in}\quad\Omega$$

$$\mathbf{x}\to\mathbf{r}^-:\ h=0,\quad \partial_\nu h=\lambda,$$

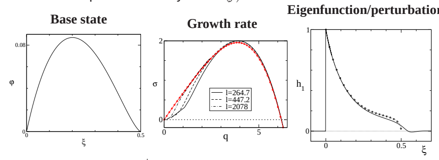
$$h^2\partial_\nu\triangle h-V_\nu^-h=0$$

$$\mathbf{x}\to\mathbf{r}^+:\ h\sim\left(-8V_\nu^+/3\right)^{1/2}\left((\mathbf{r}^+-\mathbf{x})\cdot\nu\right)^{3/2}$$



Note: $\Gamma^\pm$ are parametrized by $x=\mathbf{r}^\pm(y)$.

Base state / **Growth rate** / **Eigenfunction/perturbation**



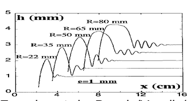
For given $l=2\pi/k$, let t^* be the time when $\max_x h_1(x,t)$ is maximal.

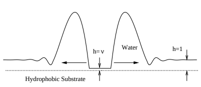
$\Rightarrow\quad \sigma(t^*)=0\quad\Rightarrow\quad\frac{2\pi}{l}w(t^*)=qc\,\frac{1}{2}\Rightarrow\quad\frac{l}{w(t^*)}=\frac{4\pi}{qc}\approx 2.09.$

Numerical results for full model: $l/w(t^*)$ indeed ≈ 2.09. (Münch/Wagner, [Physica D, 2005], King/Münch/Wagner, [J. Eng. Math. 2008])

Experimental results: $\frac{l}{w(t^*)}\approx 2.1\pm0.2$ (Gabriele et al. [PRL 2006])

Shocks in Inertial Dewetting





Experiments by Buguin/Vovelle/Brochard-Wyart, [PRL 1999]

Strong slip equation with gravity & inertia in mass/momentum formulation ($q=uh$):

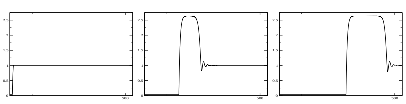
$$h_t+q_x=0,$$

$$q_t+\left(\frac{q^2}{h}+\frac{1}{2\mathrm{Fr}^2}h^2+\Psi(h)\right)_x=4S\left(h\left(\frac{q}{h}\right)_x\right)_x+\left(hh_{xx}-\frac{1}{2}h_x^2\right)_x-\frac{q}{\beta h}.$$

We focus on infinite slip, i.e. $1/\beta=\infty$, equation is in divergence form!

Numerical simulation

$\mathrm{Fr}=\sqrt{10}$, $S=0.1$, $\nu=0.04$.



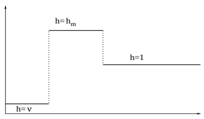
Plots show $h(x,t)$ for $t=0$, 200, 400.

System of Hyperbolic Conservation Laws

$$h_t+q_x=0,$$

$$q_t+\left(\frac{q^2}{h}+\frac{1}{2\mathrm{Fr}^2}h^2+\Psi(h)\right)_x=0,$$

Ψ associated with ϕ.



Characteristic speed c: Let $\hat x=x-ct$;
consider $h(x,t)=h_0+\delta\hat h(\hat x)$, $q(x,t)=q_0+\delta\hat q(\hat x)$, $0<\delta\ll1$

$$(A-cI)\begin{bmatrix}h_x\\q_x\end{bmatrix}=0,\quad\text{with}\quad A=\begin{bmatrix}0 & 1\\ -\frac{q_0^2}{h_0^2}+\frac{h_0}{\mathrm{Fr}^2}+\phi''(h_0)h_0 & 2\frac{q_0}{h_0}\end{bmatrix}.$$

$$\Longrightarrow\quad c_{1,2}=\frac{q_0}{h_0}\pm\sqrt{\left(\frac{1}{\mathrm{Fr}^2}+\phi''(h_0)\right)h_0}$$

Left, middle, right state

$(h,q)=(\alpha\nu,0):\quad c_2^{(l)}\sim\nu^{-1/2},\qquad c_1^{(l)}\sim-\nu^{-1/2},$

$(h,q)=(h_m,q_m):\quad c_2^{(m)}=\frac{q_m}{h_m}+\frac{\sqrt{h_m}}{\mathrm{Fr}},\qquad c_1^{(m)}=\frac{q_m}{h_m}-\frac{\sqrt{h_m}}{\mathrm{Fr}},$

$(h,q)=(1,0):\quad c_2^{(r)}=\frac{1}{\mathrm{Fr}},\qquad c_1^{(r)}=-\frac{1}{\mathrm{Fr}}.$

Classification of Shocks

Rankine-Hugoniot condition for right shock:

$$q_m-s_r(h_m-1)=0,$$

$$\frac{q_m^2}{h_m}+\frac{1}{2\mathrm{Fr}^2}(h_m^2-1)-s_rq_m=0,$$

$$\Longrightarrow\quad h_m=1+\xi,\quad s_r=\frac{1}{\mathrm{Fr}}\sqrt{1+\frac{3}{2}\xi+\frac{1}{2}\xi^2}\to1/\mathrm{Fr}=c_2^{(r)}\quad\text{for}\quad\xi\to0.$$

$\Longrightarrow$ 2-Shock!

Check Lax condition: $c_2^{(r)}<s_r<c_2^{(m)}$ i.e. Lax shock.

Rankine-Hugoniot condition for left shock:

$$q_m-s_l(h_m-\alpha\nu)=0,$$

$$\frac{q_m^2}{h_m}-\frac{1}{2\mathrm{Fr}^2}(h_m^2-\alpha^2\nu^2)+\Psi(h_m)-\Psi(\alpha)-s_lq_m=0,$$

$$\Longrightarrow\quad h_m=\alpha\nu+\eta,\quad q_m=s_l\nu,$$

$$s_l=\sqrt{\frac{(\alpha\nu+\eta)(2\alpha\nu+\eta)}{2\alpha\nu\mathrm{Fr}^2}+\frac{(\Psi(h_m)-\Psi(\alpha))(\alpha\nu+\eta)}{\eta\alpha\nu}},$$

For $\eta\to0$: $s_2\to c_2^{(l)}$ $\Longrightarrow$ again a 2-shock.
Lax condition violated, i.e. undercompressive shock:

$$c_2^{(m)}=s_l+\frac{\sqrt{h_m}}{\mathrm{Fr}}+O(\nu)>s_l$$