

SPECTRAL PROPERTIES, REGULARITY AND OPTIMAL BOUNDS FOR SOLUTIONS OF ELLIPTIC BOUNDARY VALUE PROBLEMS

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ABSTRACT. In this talk we survey a number of papers listed below due to Hans Weinberger together with some of his many collaborators during the period from 1953 to 1966 (with the exception of [11]) that are concerned with solutions of elliptic boundary value problems. We have divided these into three topics; (i) pointwise bounds and regularity at the boundary for second order elliptic equations, (ii) spectral properties of second order elliptic operators including a discussion of upper and lower bounds for eigenvalues, and (iii) inequalities and optimal bounds for other physical quantities such as capacity and torsional rigidity.

Needless to say, the list of Professor Weinberger's papers as well as his contributions to these topics is far more extensive than those we consider here.

The talk will not discuss any one paper in the list in great detail. Rather we restrict our attention to those results that we find particularly elegant, insightful and in the best tradition of the intersection of classical applied mathematics with real analysis.

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