Introduction to JPlex

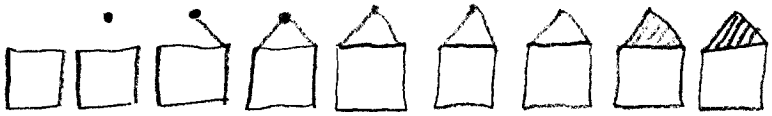
① Filtered simplicial complexes (streams)

Two rules:

- At each filtration time, you have a simplicial complex.
- The simplicial complexes increase with time.

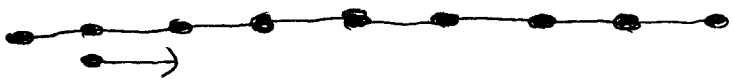
Two examples:

"No walls"

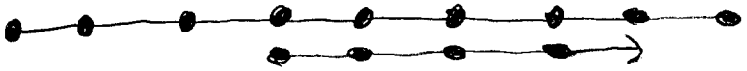


Filtration time: 0 1 2 3 4 5 6 7 8

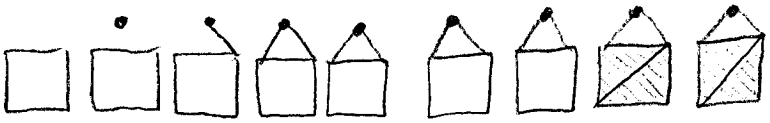
$Betti_0$ barcode:



$Betti_1$ barcode:

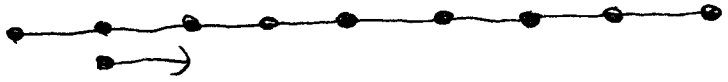


"No roof"

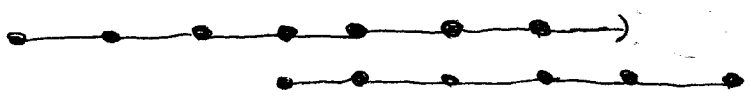


Filtration time: 0 1 2 3 4 5 6 7 8

$Betti_0$ barcode:



$Betti_1$ barcode:



Note that the two house streams have the same Betti numbers at all filtration times. However, functoriality distinguishes the streams.

② Application: data analysis

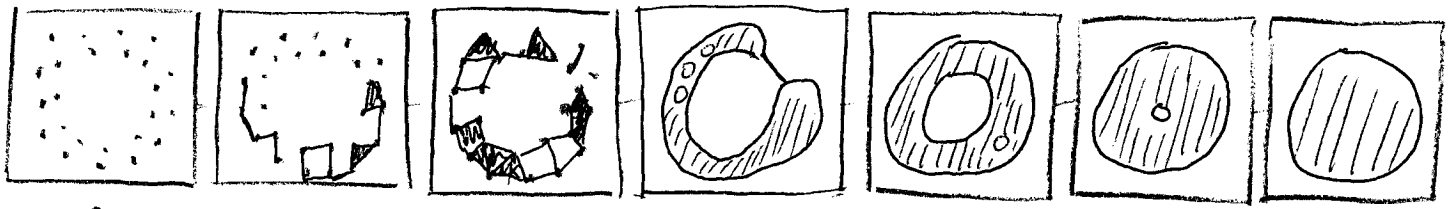
Motivating problem is "What space is this?"

Answer is "It's a circle."
How do you see it's a circle?
Plot in 2D or 3D and squint your eyes.

But what if you have a point cloud in 25-dimensional space? What if you have a point cloud in a non-Euclidean space? These point clouds are hard to visualize.

One idea is to build a filtered simplicial complex on the point cloud, and to compute its persistent homology.

Example: consider below the Vietoris-Rips (or witness or lazy witness) filtered simplicial complex on our circular point cloud data.



Betti$_0$ barcode:

Betti$_1$ barcode:

We consider short intervals to be noise, and long intervals to correspond to significant ↪

topological features. We see one long $Betti_0$ and one long $Betti_1$ interval above, which corresponds to the homology of a circle.

Remember that this process (implemented in JPlex) is an algorithm, not magic. For instance, convince yourself that you should threshold the following point cloud by density before building a filtered simplicial complex and computing its persistent homology.