Computer Algebra System SINGULAR 3-0

http://www.singular.uni-kl.de

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1 Introduction

SINGULAR 3-0 is a Computer Algebra system for polynomial computations with emphasis on the special needs of commutative algebra, algebraic geometry, and singularity theory.

SINGULAR's main computational objects are ideals and modules over a large variety of baserings. The baserings are polynomial rings or localizations thereof over a field (e.g., finite fields, the rationals, floats, algebraic extensions, transcendental extensions) or quotient rings with respect to an ideal. It can also work with some important non-commutative algebras: G-algebras. The range of applicability comprises commutative and noncommutative polynomial computations.

SINGULAR is free software; you can redistribute it and/or modify it under the terms of the GNU General Public License as published by the Free Software Foundation (version 2 of the License).

The 2004 *Richard D. Jenks Memorial Prize for Excellence in Software Engineering for Computer Algebra* was awarded to the SINGULAR team at the International Symposium on Symbolic and Algebraic Computation (ISSAC) in Santander (Spain).

2 Key Strengths of SINGULAR

- available for most hardware and software platforms
- a wide variety of choices of ground fields
- factorization of polynomials over various ground fields
- various Gröbner basis algorithms (well–orderings)
- standard basis algorithm (local and mixed orderings)
- primary decomposition for ideals and modules
- resolution of singularities (`resolve.lib`)
- noncommutative subsystem PLURAL
- dynamic extendability of functionality by means of modules written in C/C++
- additional functionality: more than 70 libraries available
- exhaustive documentation of more than 1100 pages (available in various formats)
- support via e-mail and online SINGULAR Forum

3 Absolute Factorization And Absolute Primary Decomposition

The library `absfac.lib` (by W. Decker, G. Lecerf, G. Pfister) uses Trager's idea to compute an absolutely irreducible factor by factorizing over some finite extension field L (which is chosen such that $V(f)$ has a smooth point with coordinates in L). Then a minimal extension field is determined making use of the Rothstein-Trager partial fraction decomposition algorithm. See Cheze and Lecerf, Lifting and recombination techniques for absolute factorization.

These routines are used in `primdec.lib` to compute an absolute primary decomposition: `absPrimdecGTZ` combines them with the algorithm of Gianni/Trager/Zacharias to return an list of the absolute prime components together with its number of conjugates.

4 Dynamic Modules in SINGULAR

SINGULAR Library	Dynamic Module
SINGULAR syntax	C/C++
system independent	hardware and OS specific
interpreter (slow)	compiler (fast)
Syntax errors found at run time	Syntax errors found at compile time
access to other libraries possible	access to the SINGULAR kernel possible

5 Resolution of singularities (Desingularization)

The libraary `resolve.lib` implements a variant of the algorithm of Villamayor to compute the resolution of singularities. `blowUp(J,C[,W,E])` computes the blowing up of the variety V(J) inside V(W) in the center V(C).

Applications include the intersection matrix of exceptional divisors, spectral numbers of hypersurface singularities and the Denef-Loeser Zeta function.

6 Noncommutative Rings in SINGULAR:PLURAL

Let $\mathbb{K}$ be a field and $R = \mathbb{K}[x_1, \ldots, x_n]$ be a commutative ring.
Consider an algebra $A = \mathbb{K}\langle x_1, \ldots, x_n \mid \forall\, i < j\ x_j x_i = c_{ij} x_i x_j + d_{ij}\rangle$,
where $c_{ij} \in \mathbb{K} \smallsetminus \{0\}$ and $d_{ij} \in R$, $\forall\ 1 \le i < j \le n$.
It is called **a** G**–algebra** (in n variables) if the following conditions hold:

1) $\exists \prec$, a well–ordering on R such that $\forall\ i < j \quad \mathrm{lm}(d_{ij}) \prec x_i x_j$,

2) *Nondegeneracy conditions* are fulfilled, that is $\forall\ 1 \le i < j < k \le n$

$$c_{ik}c_{jk} \cdot d_{ij}x_k - x_k d_{ij} + c_{jk} \cdot x_j d_{ik} - c_{ij} \cdot d_{ik} x_j + d_{jk} x_i - c_{ij} c_{ik} \cdot x_i d_{jk} = 0.$$

Let A be a G–algebra in n variables. Then

- A has a PBW basis $\{x_1^{\alpha_1} x_2^{\alpha_2} \ldots x_n^{\alpha_n} \mid \alpha_i \in \mathbb{N} \cup \{0\}\}$,
- A is left and right Noetherian,
- A is an integral domain with $\mathrm{gl.dim}\, A \le n$.

A GR**–algebra** is the factor algebra of a G–algebra by a proper two–sided ideal.

▷ left and two–sided Gröbner basis of a module w.r.t. any well–ordering

▷ basic ideal and module arithmetics (elimination, intersection, kernel of a homomorphism, syzygy etc)

▷ free resolutions ("normal" and "minimized" algorithms)

▷ Betti numbers for graded modules over graded algebras

▷ opposite and enveloping algebras, "opposing" given object

- center of an algebra, centralizer of a polynomial (`center.lib`)
- central character decomposition of a module (`ncdecomp.lib`)
- preimage of an ideal under a morphism of GR–algebras
- annihilators of finite–dimensional modules

7 New and Revised Functionality in SINGULAR

Ringlists

With the function `ringlist` one can represent an object of type `ring` via an object of type `list`. Thus, creating and modifying commutative and noncommutative rings becomes much easier.

New SINGULAR libraries

There are many new libraries in SINGULAR 3-0. Among them are:

`absfact.lib`	absolute factorization for characteristic 0
`control.lib`	algebraic analysis tools for Control Theory
`gmssing.lib`	Gauss-Manin connection of a singularity
`grwalk.lib`	Gröbner and Pertubation walk
`resolve.lib`	resolution of singularities
`sheafcoh.lib`	cohomology of sheaves and Tate resolution

Besides, the functionality of some existing libraries was improved and extended by implementing new algorithms.