GRÖBNER BASES AND INTEGER PROGRAMMING: PROBLEM SESSION I

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Let $\mathbf{coin} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 5 & 10 & 25 \end{bmatrix}$ and $\mathbf{3by5} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$

(1) By hand, compute $\Delta_{(5,6,2,5)}$ for **coin**. From this, describe the optimal solution set $\mathcal{O}^{\mathrm{LP}}_{(5,6,2,5)}$ for **coin**.

(2) Pick a $\mathbf{c} \in \mathbb{R}^4$ such that $(2, 0, 0, 1)$ is an optimal solution to $\mathrm{LP}_{\mathbf{coin},\mathbf{c}}$.

(3) Compute the triangulations $\Delta_{(3,0,0,1,0)}$ and $\Delta_{(3,0,0,-1,0)}$ for **3by5**. In each case, give an example of an element and a non-element of $\mathcal{O}^{\mathrm{LP}}_{\mathbf{c}}$.

(4) In your `4ti2` directory, create the text file `3by5` with input

```
3 5
1 1 1 1 1
0 1 0 1 2
0 0 1 1 2
```

Now ask for the circuits of the matrix by entering the command `./circuits 3by5` – this will create the output file `3by5.cir`. Let $\mathbf{c} = (3, 0, 0, 1, 0)$.

(a) Confirm that $(2, 1, 1, 0, 1)$ is a feasible solution for $\mathrm{LP}_{\mathbf{3by5},(3,0,0,1,0)}(5, 3, 3)$.

(b) Is $(2, 1, 1, 0, 1)$ optimal ? If not, use `3by5.cir` to find the optimal solution.

(5) Let $\mathbf{c} = (3, 0, 0, 1, 0)$ and create the text file `3by5.cost` with input

```
1 5
3 0 0 1 0
```

Now ask for a Gröbner basis (w.r.t. $(3, 0, 0, 1, 0)$) of the toric ideal of **3by5** by entering the command `./groebner 3by5` – the output file `3by5.gro` is returned. From `3by5.gro` find the initial ideal $\mathrm{in}_{(3,0,0,1,0)}(I_A)$ and, from this, determine if $(2, 1, 1, 0, 1)$ is an optimal solution for $\mathrm{IP}_{\mathbf{3by5},(3,0,0,1,0)}(5, 3, 3)$. If not, use `3by5.gro` to find an optimal solution.

(6) Same as previous question except now let $\mathbf{c} = (3, 0, 0, -1, 0)$