

Mathematical Techniques for Pre-conceptual Design

Mathematical Modeling in Industry XI
IMA

University of Minnesota

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Motivation

- Examining the process of Pre-Conceptual Design
- We want to explore the state space, not just find “optimum”
- Want (near) real time results
- Want to create a simple model that is easy to modify
- ~ 80% fidelity is okay
- Context: subsonic unmanned reconnaissance
- Discrete effects?

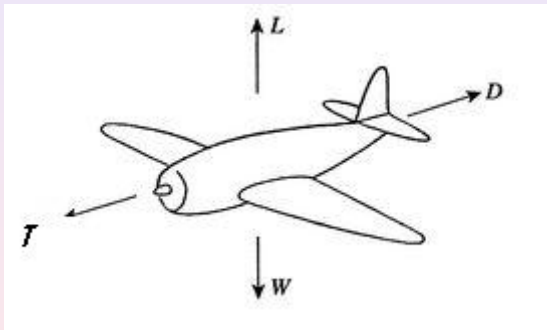
Design Variables

- Gross takeoff weight (w_0), in lbs
- Wing span (b), in ft
- Wing reference area (S_{ref}), in sq ft
- Wetted surface area (S_{wet}), in sq ft
- Engine horsepower (bhp), in hp
- Altitude (h), in ft
- Payload weight (w_p), in lbs

Performance metrics

- Range (R), in nautical miles
- Endurance (E), in hours
- Fuel available (w_f lbs), in lbs
- Maximum speed (V_m), in knots
- Cruising speed (V_c), in knots
- Stall speed (V_s), in knots

Key concept - level flight



Weight model

Total weight:

$$w_0 = w_e + w_f + w_p$$

Empty weight:

$$w_e = w_{eng} + w_{struct}$$

Empirical curve fit shows

$$w_e = a w_0^{0.91}$$

$a = 1.15$: empty weight fraction coefficient

Thus

$$w_f = w_0 - a w_0^{0.91} - w_p$$

Lift model

$$W = L$$

$$W_0 = q S_{ref} C_L$$

$$q = \frac{1}{2} \rho V_c^2$$

$$\rho = (0.0023)(1 - 6.865 h 10^{-6})^{4.256}$$

$$C_L = \frac{(0.9)w_0}{S_{ref} q}$$

Assume cruise wing loading is 90% of the takeoff wing loading

$$C_L = C_L(q) = \frac{(0.9)w_0}{q S_{ref}}$$

Drag model

$$D = D(q) = qS_{ref}(C_{D_0} + KC_L^2)$$

$$C_{D_0} = c_{fe} \frac{S_{wet}}{S_{ref}}$$

$$K = \frac{(0.424)S_{ref}}{b^2}$$

C_{D_0} : Parasitic Drag (skin friction)

KC_L^2 : Drag due to lift

(c_{fe}) : Skin Friction Coefficient = .005

Thrust model + Cruise Speed

Assume engine uses 75% power in cruise

$$T = D$$

$$\frac{(0.75)(550 \text{ bhp } \eta_p)}{V_c} = q S_{ref} (C_{D_0} + KC_L^2(q))$$

Where propellor efficiency (η_p) = .75

Solve implicit equation for V_c (note: is converted to knots here)

$$(4.069)\rho C_{fe} S_{wet} V_c^4 - (928.95) \text{ bhp } \eta_p V_c + \frac{(0.687)w_0^2}{\rho b^2} = 0$$

Range + Endurance

Breuet Range Equation - derived from integrating fuel consumption and weight

$$R = \frac{550\eta_p}{c_{bhp}} \frac{L}{D} \ln \left(\frac{(0.975) w_0}{w_0 - w_f} \right) \quad ft$$

We care about Endurance

$$E = \frac{R}{V_c}$$

Fuel consumption coefficient implicitly defined

$$c_{bhp} \cdot bhp \cdot E(c_{bhp}) \cdot 3600 = w_f$$

Stall speed

Approach speed is closely related to stall speed

Stall occurs at the maximum lift coefficient, $C_{L_{max}}$

($C_{L_{max}} = 1.4$ for wing types we're using)

$$w_0 = q_s S_{ref} C_{L_{max}}$$

$$w_0 = \frac{1}{2} \rho V_s^2 S_{ref} C_{L_{max}}$$

$$V_s = \frac{1}{1.689} \sqrt{\frac{2 w_0}{\rho S_{ref} C_{L_{max}}}} \quad \text{in knots}$$

Sanity Check!

We have a model - does it produce reasonable results?
Test case: NASA Hunter UAV



Sanity check, continued

Our model	Actual Data
$V_c = 90$ kts	$V_c = 100$ kts
$R = 1188$ n.mi.	$R = 288$ n.mi.
$E = 13.2$ hr	$E = 12$ hr

Issue - we have incomplete/conflicting data, so we're not sure.

Constraints

Physical constraints:

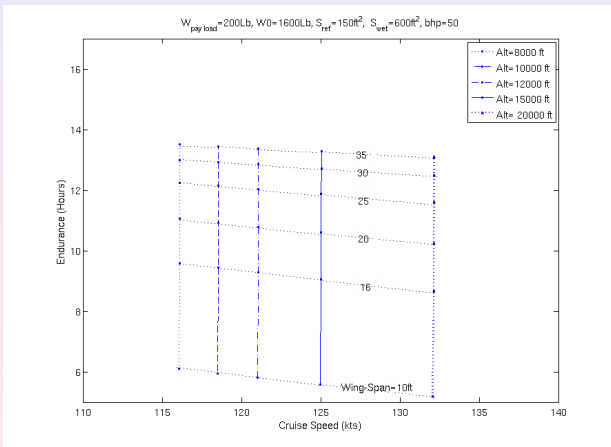
$$V_s \leq V_c \leq \text{Mach } 0.8$$

$$C_L \leq C_{L_{max}}$$

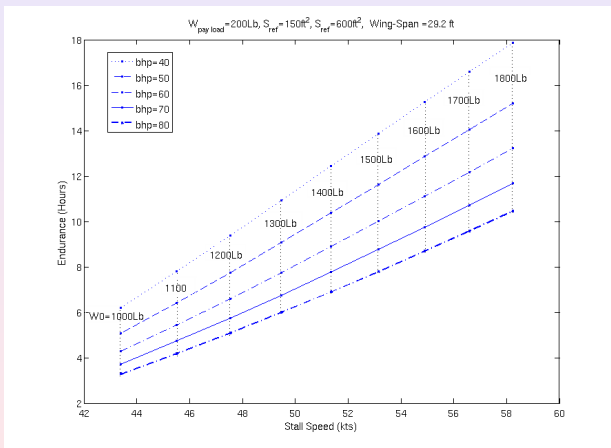
$$2 \leq \frac{S_{wet}}{S_{ref}} \leq 8$$

Trade space

Model is deterministic - can plot quickly



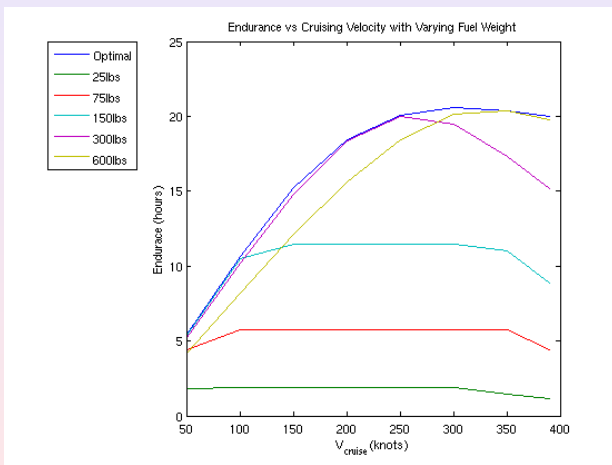
Trade Space II



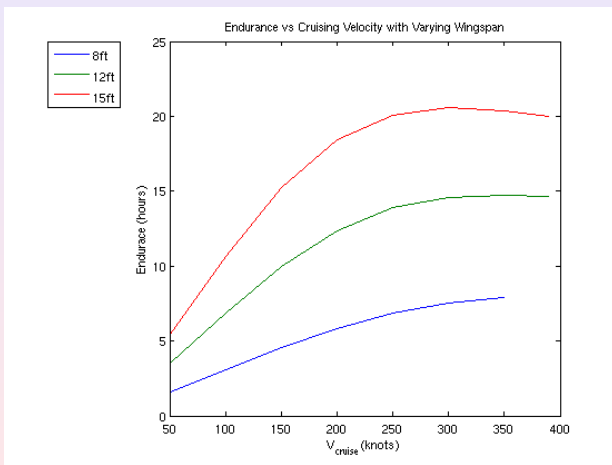
Optimized trade space

- Add constraints of our choosing
- e.g. fixed speed for picture taking
- Can find optimum configurations based on this
- Effects of another constraint

Constrained trade study



More optimized examples



Outer Approximation

- Breaks a MINLP into MILP and NLP.
- Given initial state, solves NLP for discrete part
- Linearizes the problem, then solves a MILP
- iterates until converges

Problem: Only feasible for convex functions

Branch and Bound

- Breaks discrete variable states into a binary format
- At each iteration, we solve 2 NLPs for $x_i = 0, 1$
- Other x_j are continuous
- Check result - continue where we have seen smallest value of objective function

Problem: Can be very efficient ($O(n)$) or very not ($O(2^n)$)

BONMIN (what we're using)

- Uses branch and bound for discrete part
- Uses primal-dual interior point method with BGFS for NLP
- Solves our NLP efficiently
- Creates earlier graphs in about 3-4 minutes

FLOPS

- Calculates/Estimates weight and wetted surface areas using geometry
- Uses physics and experimental data to generate drag, lift, thrust data at all (relevant) altitudes, speeds, and weights
- Data is used to integrate equations of motion during the mission to determine performance

Advantages of FLOPS

- Can model a wide variety of aircraft
- Models more than just plane structure - can design engines
- Allows more geometries - larger design space
- Much more accurate results

Disadvantages of FLOPS

- Slower than our method, though not by a *huge* amount (15-20 minutes to generate a similar plot)
- No consideration of stall speed (explicitly, at least)
- Difficult to extend to other constraints
- Adding discrete effects gives another layer of complexity to an already complex method

Recap

- Developed a simple model - yes
- Developed a good model - maybe? Need better data
- Investigated extendable tools - yes
- Real time results? - not quite, but order of magnitude improvement
- Easy to apply to other missions - Framework in place, need empirical data
- Discrete effects? - no, but framework there

What next?

- Volume constraint for fuel
- Add discrete effects!
- Get better UAV data to ensure model is reasonable
- Takeoff/Landing
- Come from the other direction - can FLOPS model be tailored to these types of models?

References

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Thanks!

Questions?