Scale-space in Hyperspectral Image Analysis

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ABSTRACT

For two decades, techniques based on Partial Differential Equations (PDEs) have been used in monochrome and color image processing for image segmentation, restoration, smoothing and multiscale image representation. Among these techniques, parabolic PDEs have found a lot of attention for image smoothing and image restoration purposes. Image smoothing by parabolic PDEs can be seen as a continuous transformation of the original image into a space of progressively smoother images identified by the "scale" or level of image smoothing. The semantically meaningful objects in an image can be of any size, that is, they can be located at different image scales, in the continuum scale-space generated by the PDE. The adequate selection of an image scale smoothes out undesirable variability that at lower scales constitute a source of error in segmentation and classification algorithms. This work proposes a framework for generating a scale space representation for a hyperspectral image (HSI) using PDE methods. We illustrate some of our ideas by hyperspectral image smoothing using nonlinear diffusion.

STATE OF THE ART

- Witkin[1] (1983) introduces the Scale-Space concept: different objects appear at different image scales in the image. The scale-space of Witkin was based on the isotropic diffusion (Gaussian Blurring) of an image.
- Perona and Malik[2] (1990) present a nonlinear diffusion equation to generate a Scale-Space without blurring the edges. In fact, their intention was to enhance the image edges.
- Alvarez, et al.[3] (1993) proved that the Perona Malik diffusion equation is ill-posed, due to unstable backward diffusion on the image edges. They also show how to obtain a continuous Scale-Space, by regularizing the Perona-Malik nonlinear equation, and formalize the concept of Scale-Space as a transformation with the following properties:
  - ✓ Architectural: recursivity, causality, regularity, locality and Consistency.
  - ✓ Invariance: stability and shape-preserving.
- Sapiro and D. L. Ringach[4] (1996) presents a general framework for vector-valued images based on the Di Zenzo's[5] generalization of gradient.
- J. Weickert (1996-2002) establishes the requirements that a discretized diffusion equation must hold to constitute a scale-space with the same properties than the continuous transformation[6]. He also introduces and extends to vector-valued images, the concept of anisotropic diffusion using a tensor valued diffusion coefficient[7] and the Additive Operator Splitting as a robust semi-implicit scheme to solve the regularized nonlinear diffusion equation[8].
- A recent (2000, 2001) scale-space framework, called direction diffusion, was proposed for vector-valued images by B. Tang et al[9,10] based on the direction rather than the magnitude of the image vectors.
- D. Tschumperlé and R. Deriche[11] (2005) propose a unified anisotropic diffusion equation in terms of local filtering with spatially adaptative Gaussian kernels for vector valued images.

CONTRIBUTION

The scale-space framework is well-known and developed in computer vision for vector-valued images. Nevertheless, this framework has been limited in practice to color images and in much less degree to multispectral images, without any study on the effect of nonlinear diffusion on the accuracy of classification of hyperspectral imagery which consists of hundreds of spectral bands. Also, the high dimensionality of the data makes indispensable a reduction in the computational complexity of the scale-space analysis in hyperspectral imagery, in order to make this approach attractive for the remote sensing community.

In this work, we analyze the effect of nonlinear diffusion on the classification of hyperspectral imagery and the use of semi-implicit schemes and preconditioned conjugated gradient methods to speedup the diffusion process.

CONTINUOUS SCALE-SPACE

Given the limited experience that exists with nonlinear diffusion in hyperspectral imagery, we use in our work a simple extension of the Perona-Malik nonlinear diffusion equation to vector valued-images, given by[7]:

$$\frac{\partial \mathbf{v}(x,y,t)}{\partial t} = \frac{\partial}{\partial x}\left(g\left(f(\lambda_+,\lambda_-)\right)\frac{\partial \mathbf{v}}{\partial x}\right) + \frac{\partial}{\partial y}\left(g\left(f(\lambda_+,\lambda_-)\right)\frac{\partial \mathbf{v}}{\partial y}\right) \quad \mathbf{v}(x,y,t=0) = \mathbf{v}_0. \qquad [1]$$

Where, $\mathbf{v}(x, y, t): R^2 \times R \rightarrow R^m$ is a m-band vector-valued image at scale t, g is the diffusion coefficient, which depends on $f(l+, l-)$, a measure of the infinitesimal dissimilarity of the image around a point, $\mathbf{v}$, with $l+$ and $l-$ being the eigenvalues of the first fundamental form of $d\mathbf{v}_\sigma \cdot \mathbf{v}_\sigma$ is simply the convolution of the image with an isotropic Gaussian kernel, i.e. $\mathbf{v}_\sigma = \mathbf{v} * G(0,\sigma)$.

In particular, we use the nonlinear diffusion coefficient proposed by Weickert[8]

$$g(s) = \begin{cases} 1 & s = 0 \\ 1 - e^{-\frac{3.31488}{(s/K)^8}} & s > 0, \end{cases}$$

where, $s=f(\lambda+,\lambda-)$ and K is a threshold value for s. If $s \geq K$, the diffusion reduces significantly (near edges) and for $s < K$, the diffusion increases (within the image objects). The simplest dissimilarity metric proposed is $f(l+, l-) = l_+ - l-$, which corresponds to

$$f(\lambda^+, \lambda^-) = \lambda^+ - \lambda^- = \sqrt{\sum_{i=1}^{m} \left|\nabla v_i\right|^2}.$$

That is, the mean square value of the gradient of the image, on each spectral band.

Thanks to the pre-smoothing of the image with an isotropic Gaussian kernel, equation [1] is well-posed and generates a continuous scale-space[7].

DISCRETE SCALE-SPACE

If we number the pixels of a hyperspectral image, in major column format, the image can be represented in matrix form using a $p \times m$ matrix $\mathbf{V}$, where p is the number of pixels in the image and m the number of bands, given by

$$\mathbf{V} = \left[\mathbf{v}_1 \quad \mathbf{v}_2 \quad \cdots \quad \mathbf{v}_p\right]^T,$$

Where, $\mathbf{v}_i$ is a vector representing the i^{th} band in the image taken in major column format. If we discretize the scale as $t=n\Delta t$ and the spatial coordinates as $x=i\Delta x$, and $y=j\Delta y$, the explicit discretization of [1], in matrix notation is:

$$\mathbf{V}^{n+1} = \left(\mathbf{I} + \mu \mathbf{G}^n\right)\mathbf{V}^n, \qquad [2]$$

Where, $\mu = \Delta x \Delta y / \Delta t$. Equation [2] generates a discrete scale-space provided that $\mathbf{G}$ satisfies some properties[8] and $\mu \leq 1/4$. Nevertheless, the constrain on μ imposes a serious limitation on the applicability of the nonlinear diffusion equation on hyperspectral imagery, given that larger scale values implies a large number of iteration steps. On the other hand, semi-implicit schemes are stable for all values of μ, being limited only by the accuracy of the computed solution[8]. Equation [3] is the semi-implicit version of [2]:

$$\left(\mathbf{I} - \mu \mathbf{G}^n\right)\mathbf{V}^{n+1} = \mathbf{V}^n, \qquad [3]$$

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Using [3] we can achieve larger scale-steps at the price of having to solve a linear system of equations. Nevertheless, if we use a neighborhood of 4 pixels to discretize [1], **G** has only five diagonals and it can be factored using Additive Operator Splitting (AOS), Alternating Direction Implicit (ADI) methods or we can solve [3] using the conjugated gradient method, accelerated by using preconditioners (PCG). For more detail on the specific ADI and PCG methods used see [12,13].

EXPERIMENTS

We use in our experiments three real hyperspectral images with intensities normalized in the [0 1] range: the NW Indian Pines and Cuprite mining district images taken with the AVIRIS sensor in 1992 and 1996, respectively, and the false leaves image taken by the surface optics company using the SOC-700 hyperspectral imager to which we added 10% in amplitude of white Gaussian noise. Figure 1 shows the ground truth available for the Indian Pines and Cuprite images and Figure 2 shows the training and testing samples selected using this information.

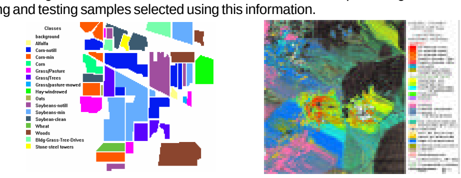

Figure 1 Ground truth a) Indian Pines, b) Cuprite image

Figure 2 Training and testing samples on a) Indian Pines image (RGB shows bands 29, 15, 12), b) Cuprite image (RGB shows bands 183, 193, 207), c) Noisy False Leaves image (RGB shows bands 90, 68, 29)

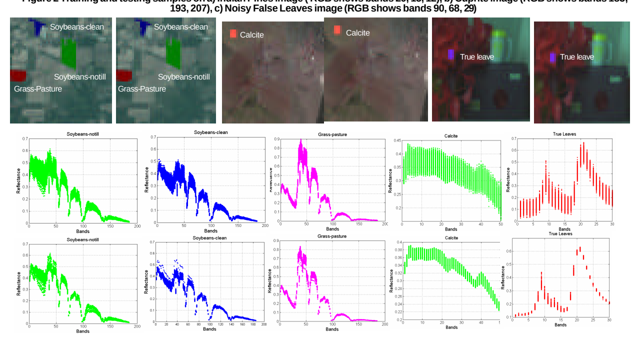

Figure 3 first row shows selected areas over the original and smoothed hyperspectral images, second row shows the superposition of the spectral signatures over the selected areas on the original images and the third row shows the superposition of the spectral signatures on the selected areas on the images at a higher scale.

Numerical method				Overall Classification Accuracy (%)					
Scheme	m/m_0	t(min)	S	ML	FLL	ED	ECHO	SAM	MF
Indian Pines									
Original	-	0.00	-	75.3	63.7	35.0	80.6	41.0	47.1
Explicit	1	3.65	1.0	89.2	89.9	50.6	90.1	50.6	74.0
AOS	50	0.09	***42.5***	80.2	85.7	40.5	93.7	44.9	62.3
Douglas	10	0.49	7.5	***94.8***	91.8	51.0	91.5	50.4	71.8
Cuprite									
Original	-	0.0	-	87.3	92.2	56.1	93.5	85.5	66.0
Explicit	1	11.38	1.0	86.2	97.1	58.4	97.1	90.6	79.1
ADI	25	1.08	***10.6***	46.9	96.9	58.8	96.9	90.6	78.7
Peaceman-Rachford	10	1.58	7.2	79.5	***97.3***	58.3	***97.3***	90.9	79.7
False Leaves									
Original	-	0.0	-	79.3	77.1	55.9	82.0	74.4	48.7
Explicit	1	13.24	1.0	47.7	93.4	60.1	94.3	78.0	73.6
ADI	20	1.07	***12.3***	41.2	93.6	60.0	94.1	77.7	74.0
PCG-AOS-Cholesky	50	1.65	8.0	49.1	***95.0***	60.0	***95.0***	77.9	73.0

This table shows ($\mu_0 = 1/4$) the best speedup (S), running time and classification accuracies achieved using the original image and smoothed images. The smoothed images where obtained using the explicit and semi-implicit methods. The classification accuracies corresponds to 6 different classifiers: Maximum Likelihood (ML), Fisher Linear Likelihood (FLL), Euclidean Distance (ED), the ECHO spectral-spatial algorithm, the Spectral Angle Mapping (SAM), and the Matched Filter (MF). It can be noticed the improvement in classification accuracy achieved by the semi-implicit methods, while achieving high speedups, relative to the explicit scheme.

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