Leslie-Ericksen

Hedgehog-Antihedgehog Annihilation to a Static Soliton

P. E. Cladis, Advanced Liquid Crystal Tech., USA

Nematic Liquid Crystals

Cylindrical Geometry

Point Defect Dynamics

Uniaxial Nematics

Orientationally ordered 3D liquids

Define **director, n**

n ⟷

unit pseudo-vector

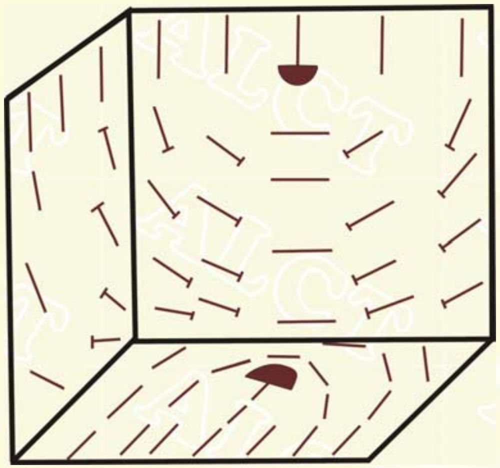
Theorem: **Only 1 line defect – the Möbius Defect (S=±1/2)**

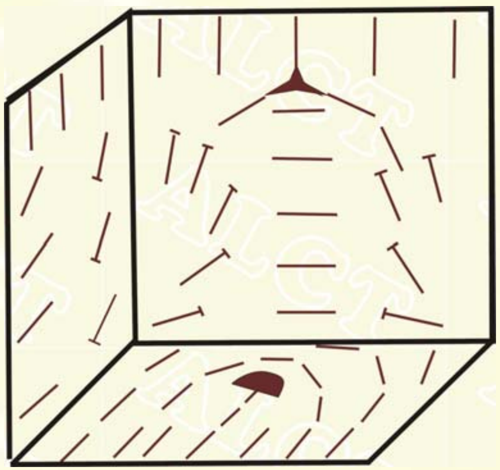
Corollary: **S=+1/2 homotopic to S=-1/2**

Corollary: **Energy $\propto S^2$**

Corollary: Point Defects

3D Möbius Defects



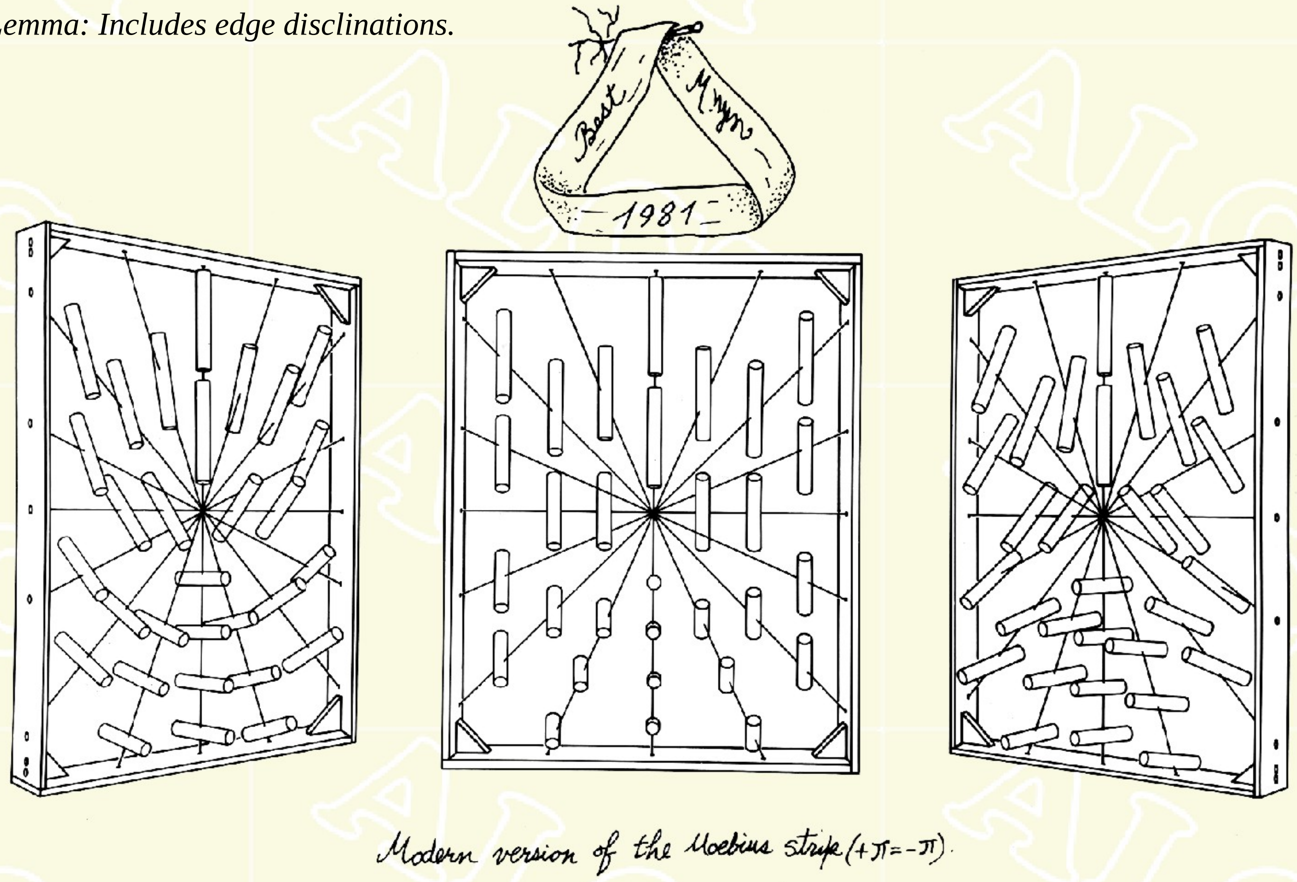


+1/2 → +1/2   +1/2 → -1/2

Y. Bouligand, P.E. Cladis, L. Liébert, and L. Strzelecki, Study of thin Films of Polymerized Liquid Crystals, Mol. Cryst. Liq. Cryst. 25, 355 (1973).

Yves Bouligand Lemma

Lemma: Includes edge disclinations.



In nematics, only 1 line defect, the Möbius line.

S=1 is homotopic to the Uniform State

$$\mathbf{n} = \begin{bmatrix} n_r \\ n_\varphi \\ n_z \end{bmatrix} = \begin{bmatrix} \cos\Phi(r) \\ 0 \\ \sin\Phi(r) \end{bmatrix}$$

Boundary condition $\rightarrow$ At $r = R$, $\mathbf{n} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Frank Free Energy

$$F = \frac{K}{2}\int_0^R dr \left[\frac{\cos^2\Phi}{r} + r\left(\frac{d\Phi}{dr}\right)^2\right]$$

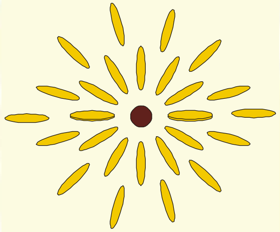
$$\frac{d}{dr}\left(r\frac{d\Phi}{dr}\right)^2 = -\frac{\sin\Phi\cos\Phi}{r} \longrightarrow \left(r\frac{d\Phi}{dr}\right)^2 = A^2 - \sin^2\Phi$$

Solutions: A=0

$$\left(r\frac{d\Phi}{dr}\right)^2 = A^2 - \sin^2\Phi$$

A is constant of integration

A=0 $\quad (\exists \text{ Trivial solution } \Phi = 0)$



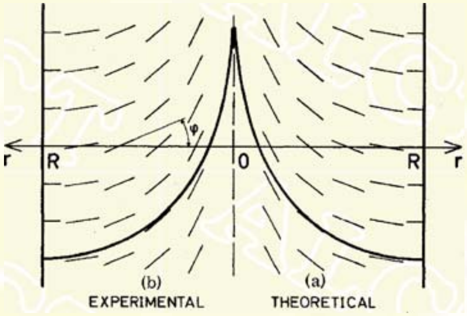
$$E_M = \pi K_1 \ell n(R/r_c) + \text{core}$$

Solutions: Energy Minimum, A=1

A=1

$$\frac{r}{R} = \cot\left(\frac{\pi}{4} + \frac{\Phi}{2}\right), \quad 0 \leq \Phi \leq \frac{\pi}{2}$$

$$\frac{R}{r} = \cot\left(\frac{\pi}{4} + \frac{\Phi}{2}\right), \quad 0 \geq \Phi \geq -\frac{\pi}{2}$$



Static Solitons

$$E_{SS} = \pi\left(2K_1 + K_3 \frac{k}{\tan k}\right)$$

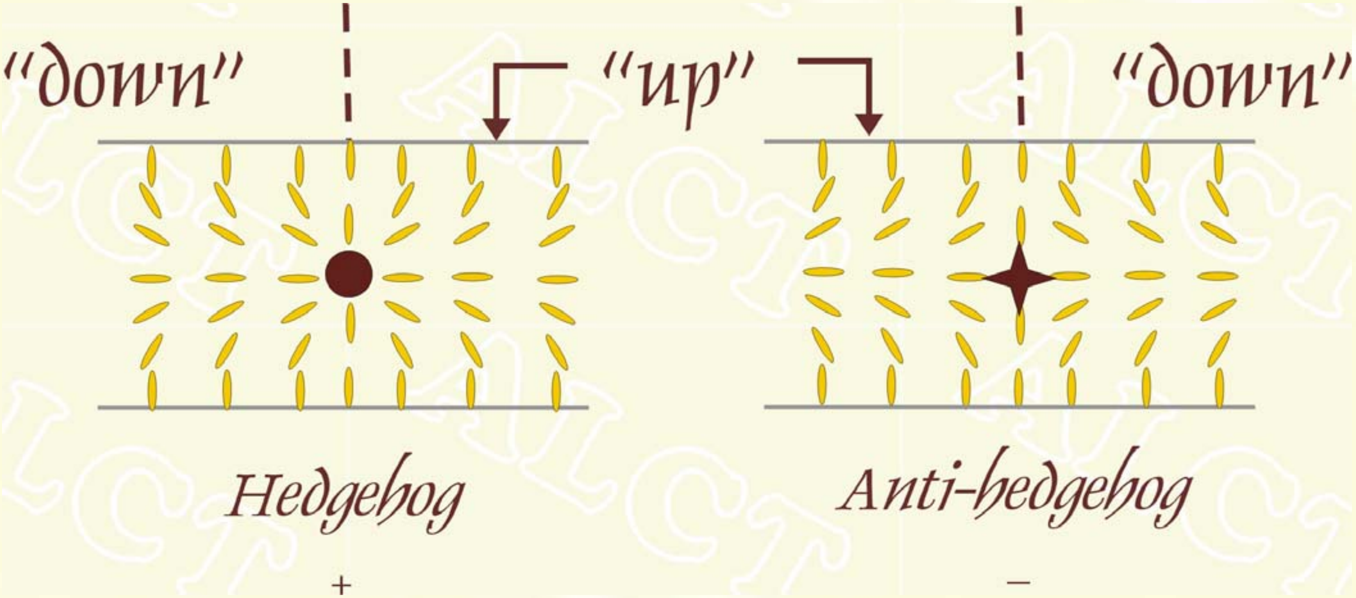
per unit length

Dresden 1999

P.E. Cladis and M. Kléman

[J. de Phys. (Paris) 33, 591 (1972)]

Asymptotic Freedom

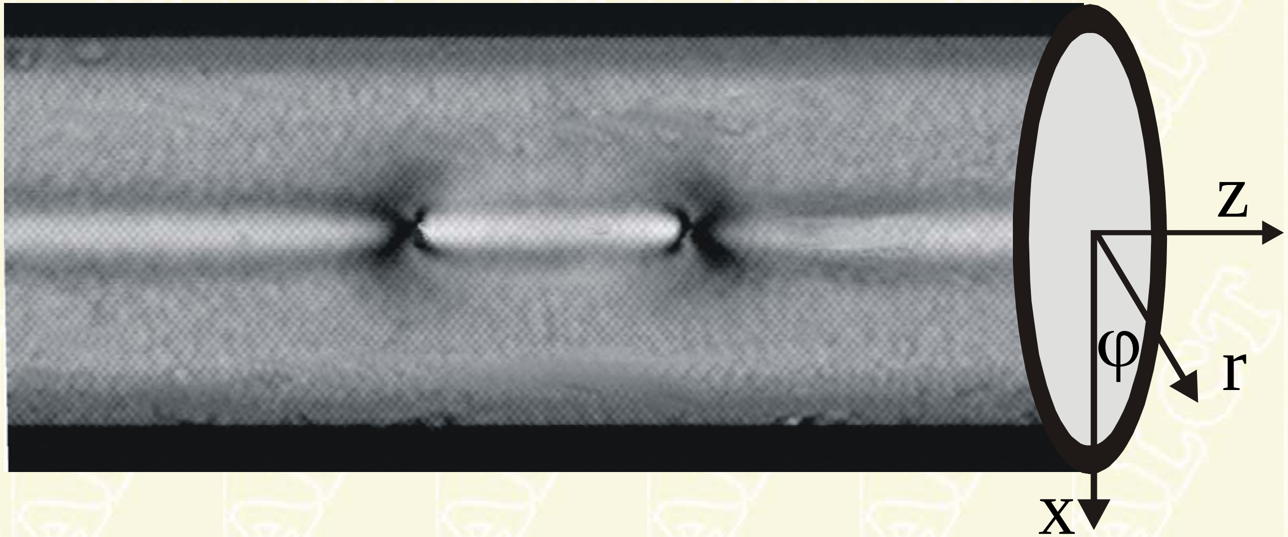


$$E_H = 8\pi K_1 R \qquad E_{\bar{H}} = \frac{8}{5}\pi R\left(K_1 + \frac{2}{3}K_3\right)$$

Spherical estimate

P.E. Cladis and H.R. Brand, Physica A 326, 322 (2003)

Point Defects

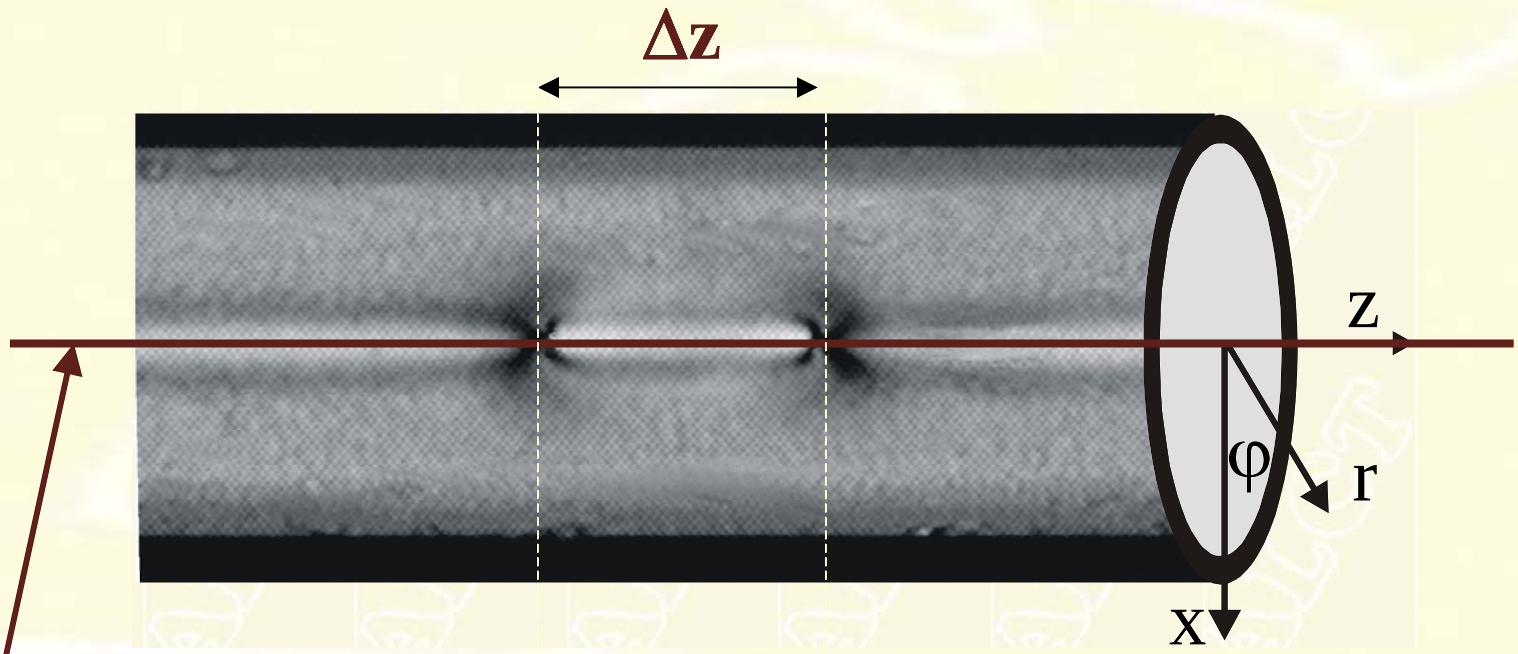


C.E. Williams, P. Pieranski and P.E. Cladis, Phys. Rev. Lett. **29**, 90 (1972)

Dresden 1999

Happy Birthday, Jerry! From all of us.

Δz-t (time) Plots

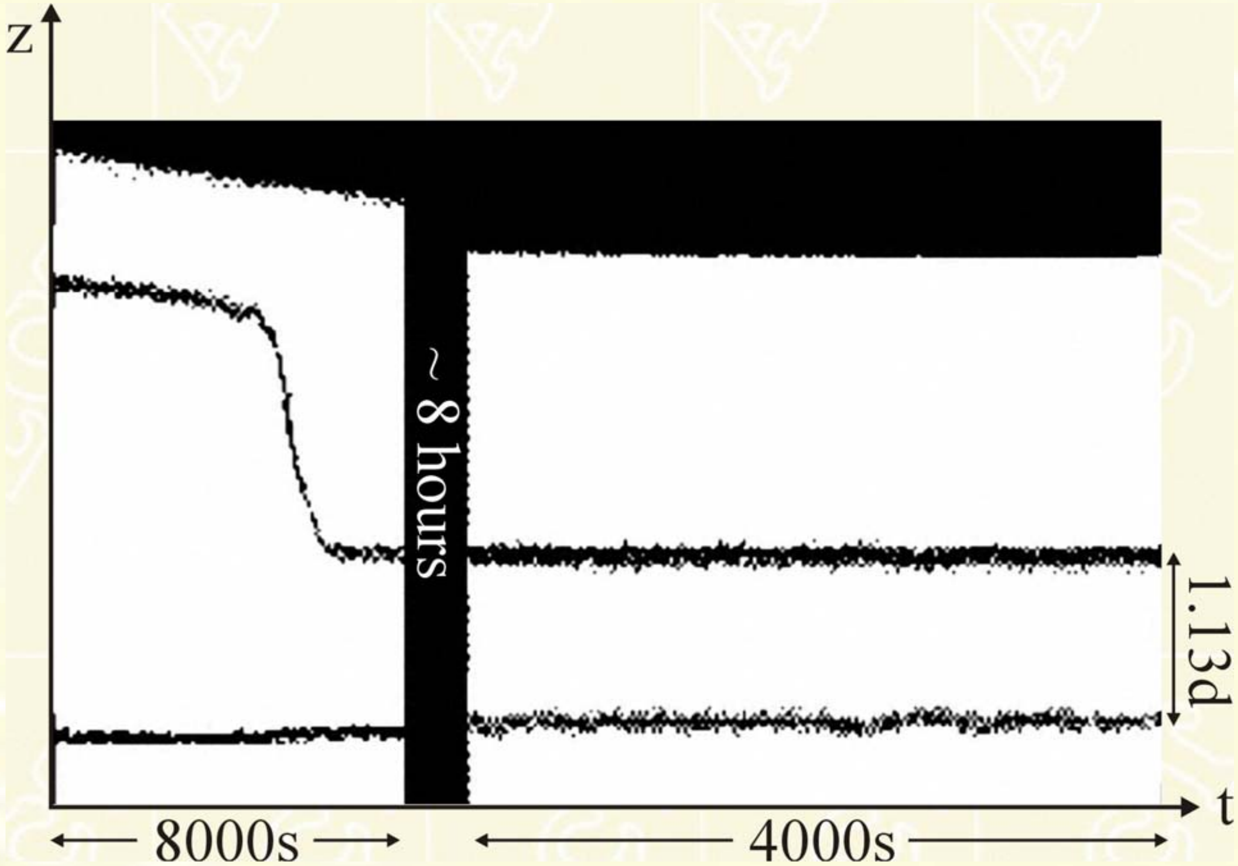


Space-time plots:

Video-line sampling

Tube diameter=d=2R

Asymptotic Freedom



P.E. Cladis and H.R. Brand, Physica A 326, 322 (2003)

Solutions: A≥1

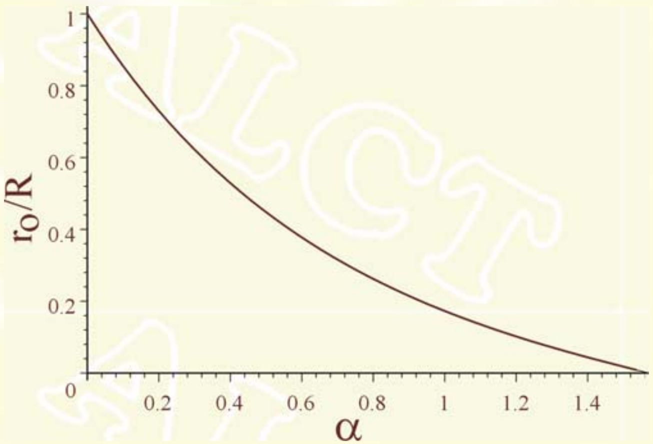
A=1/sinα

$$r/R=\exp\{-\sin\alpha\,\mathcal{F}(\varphi,\sin\alpha)\}$$

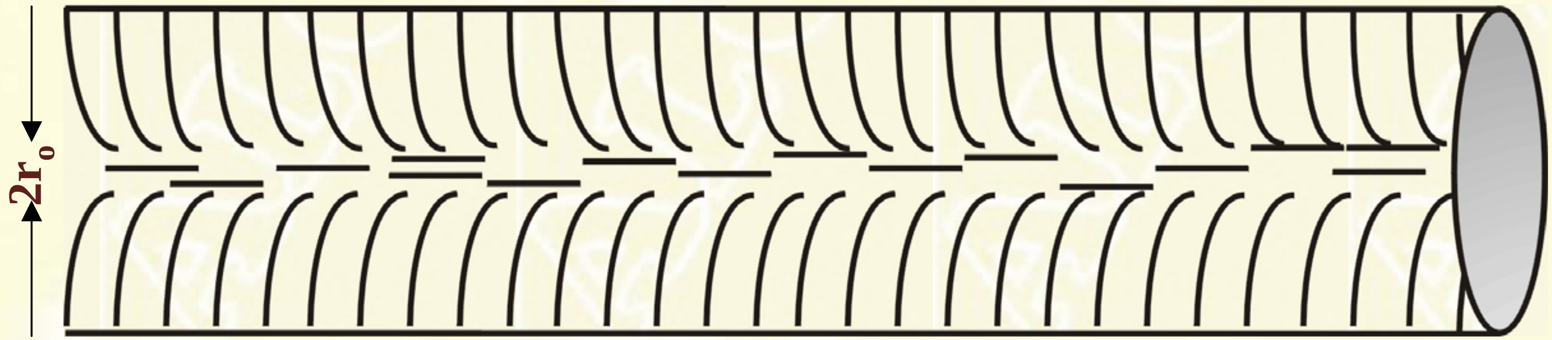
Elliptic function of the first kind

Complete

$$r_o/R = exp(-sin\alpha\mathcal{K}(\alpha)),\quad 0 \leq \alpha \leq \frac{\pi}{2}.$$

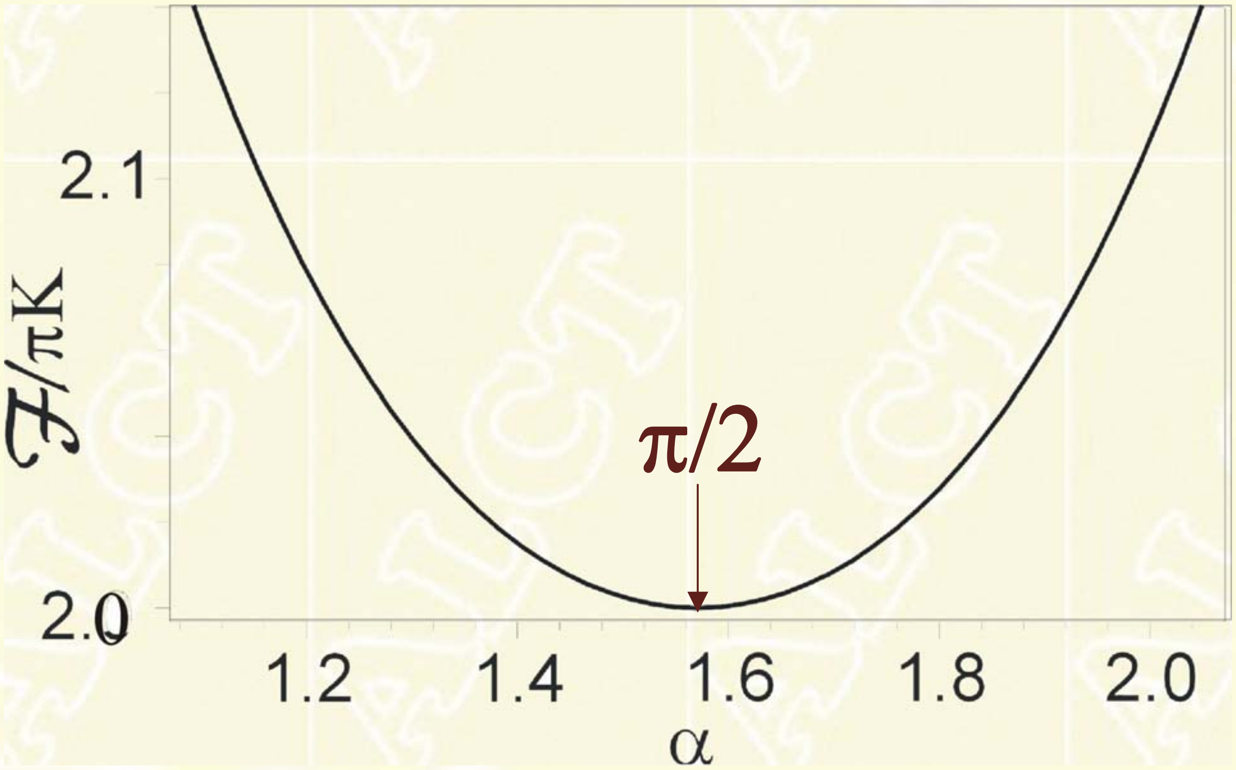


Solutions: A≥1 (continued)



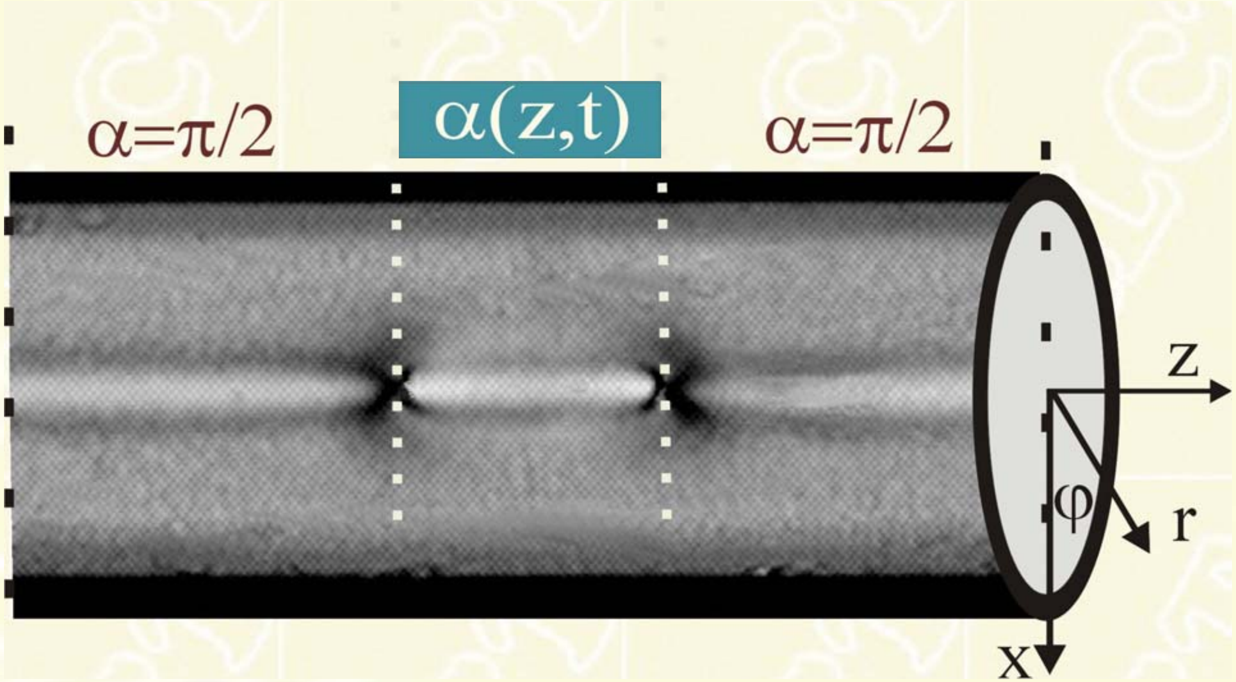
$$\frac{\mathcal{F}}{\pi K} = \frac{1}{\sin\alpha}\left\{2\mathcal{E}(\alpha) - \cos^2\alpha\,\mathcal{K}(\alpha)\right\}$$

$\mathcal{F}$ vs α☛

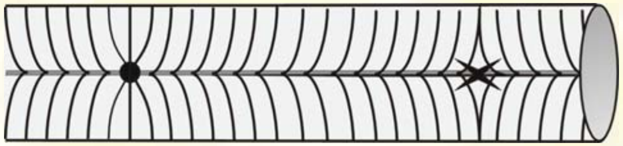
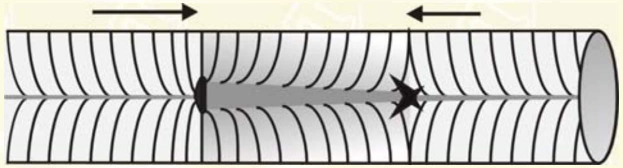
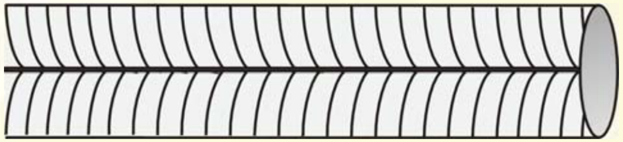


☛ : **Constant of integration A=1/sinα**

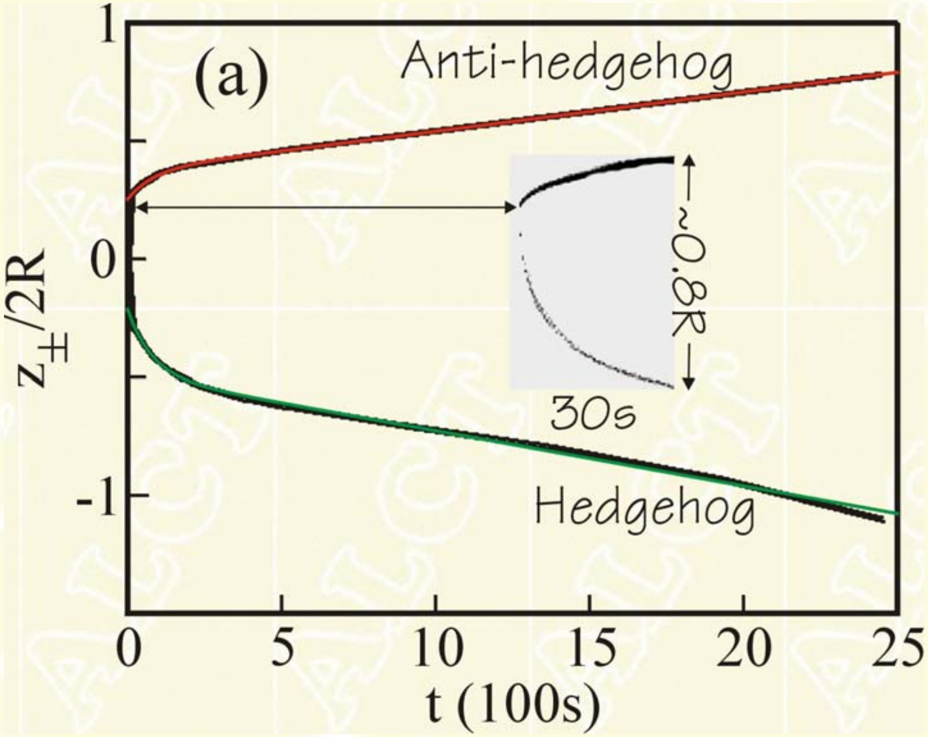
Model Dynamics with α(z,t) in region between Hedgehog and Anti-Hedgehog



MODEL

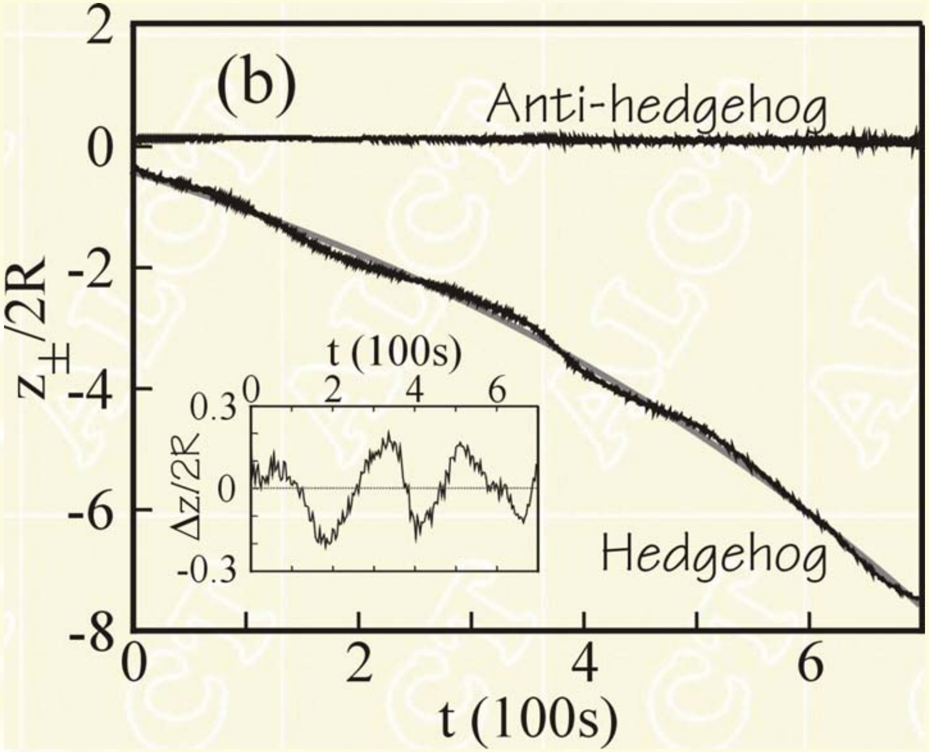
α(z,t)	Situation	Sketch	Feature
$\pi/2$	**stationary**		**Asymptotic freedom**
$\neq \pi/2$	**Out of equilibrium**		**Closing to annihilation**
$\pi/2$	**NA** **No defects**		**Annihilated**

Observed Countdown to Annihilation

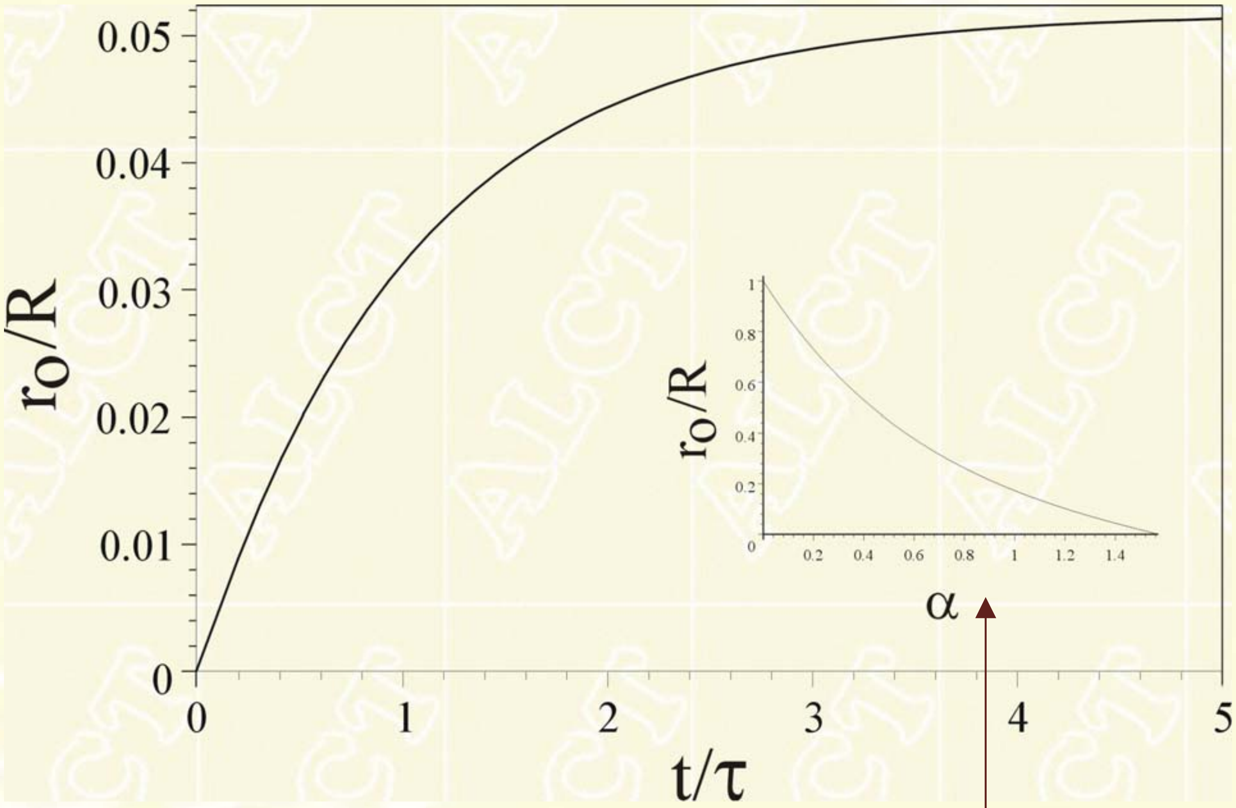


$$z = z_\pm^{(o)} + z_\pm^{(1)} exp[-t/\tau_\pm] + v_\pm t$$

+Smectic Fluctuations



Model



$$\alpha = \alpha_o + (\pi/2 - \alpha_o)exp(-t/\tau).$$

Point Defect Dynamics

-depends sensitively on perturbation magnitude

sufficiently small Ž asymptotic freedom

sufficiently large Ž 2 regimes: z/R>1 and z/R<1

-asymmetric Ž larger effective mass of antihedgehog

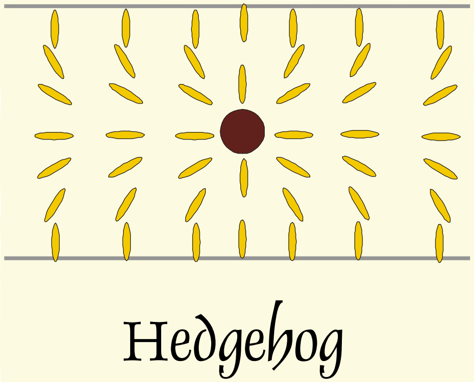
The End

Acknowledgements

Photos of Dresden 1999 Defect Meeting by **Dr. Isabelle Kraus**

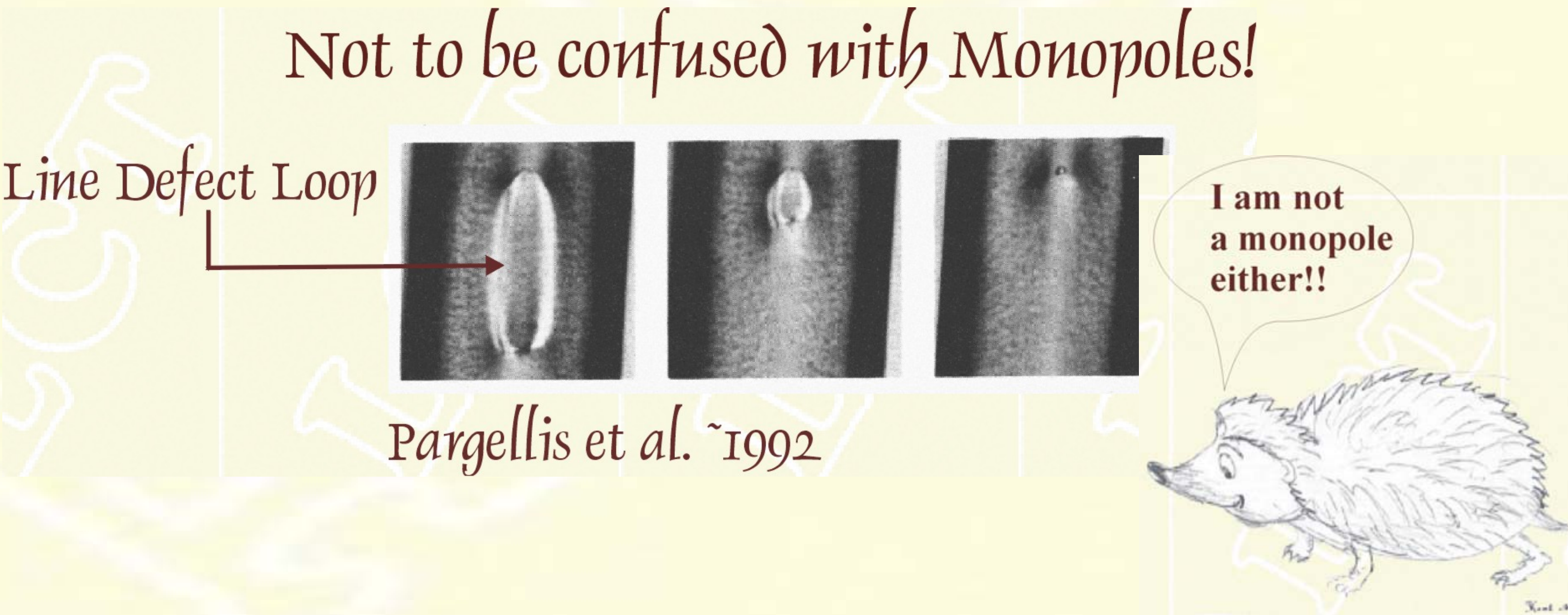
Hedgehog sketch from:
http://hem.passagen.se/hedgehogs/hedgehogs.html

Hedgehog



http://hem.passagen.se/hedgehogs/hedgehogs.html

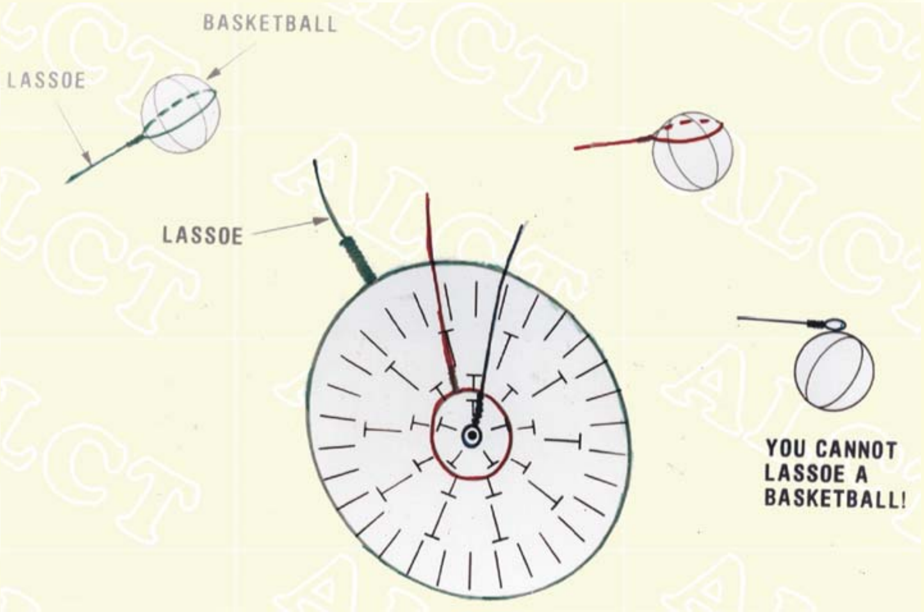
Not Monopoles



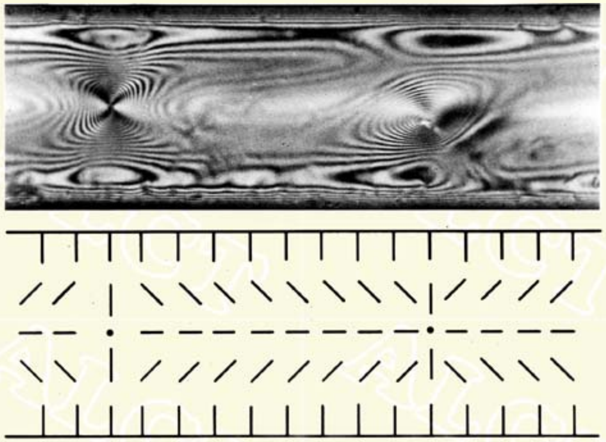
Toulouse Theorem

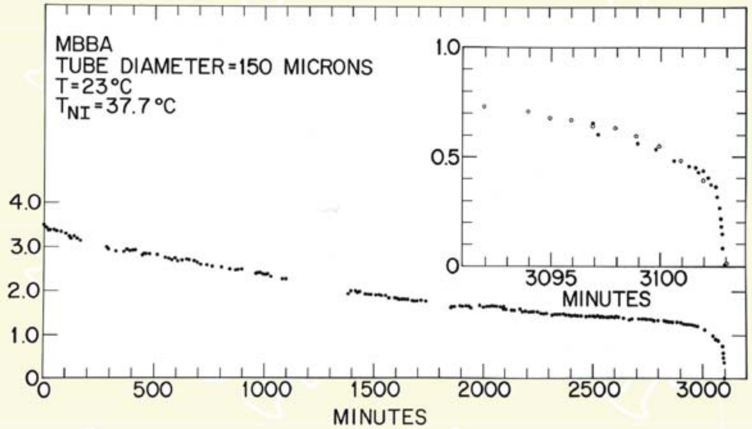
You cannot lassoe a basket ball.

Proof:

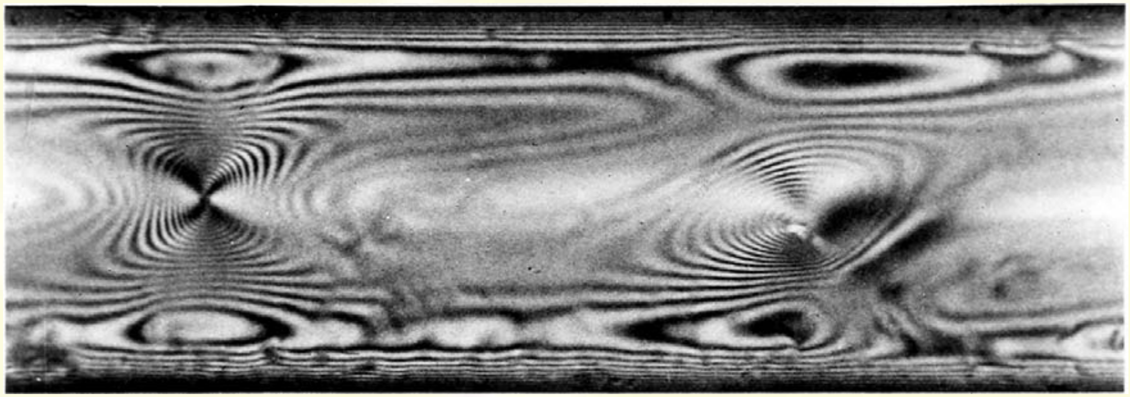


Mayola Walters and PEC (1980)





Monochromatic Scan



Pawel and PEC Dresden 1999