BOUNDARY VARIATIONS AND ANALYTIC CONTINUATION IN ELECTROMAGNETIC AND ACOUSTIC SCATTERING*

Fernando Reitich
School of Mathematics
University of Minnesota
Minneapolis, MN 55455

Joint work with Oscar Bruno (Applied Mathematics, Caltech).

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ELECTROMAGNETIC AND ACOUSTIC SCATTERING

Resonance regime: wavelength $\approx$ lengthscale of scatterer

(time-harmonic)

MAXWELL’S EQUATIONS

\[
\nabla \times \vec{E} = i\omega \mu \vec{H} \\
\nabla \times \vec{H} = -i\omega \epsilon \vec{E}
\]

with appropriate boundary conditions

\[
\vec{n} \times \left( \vec{E}^{\text{out}} - \vec{E}^{\text{in}} \right) = 0
\]

and

\[
\vec{n} \times \left( \vec{H}^{\text{out}} - \vec{H}^{\text{in}} \right) = 0
\]

Here

\[
\vec{E}^{\text{out}} = \vec{E}^0 + \vec{E}^+ \\
\vec{H}^{\text{out}} = \vec{H}^0 + \vec{H}^+
\]

HELMHOLTZ EQUATION

\[
\Delta V + k^2 V = 0
\]

\[
V^{\text{out}} = 0 \quad \text{or} \quad \frac{\partial}{\partial n} V^{\text{out}} = 0
\]

\[
V^{\text{out}} = V^0 + V^+
\]
Solution by

- Integral Equations
- Finite Elements
- Finite Differences
- ...

- Variation of Boundaries: O. Bruno/FR,
  - 2D bounded obstacles, *ACES J.*, 1996
  - Cavities and waveguides (eigenvalue problems), 1999

  **Idea:** High-order perturbations ...
ANALYTIC CONTINUATION?

\[ y = f(x) \]

\[ y = \delta f(x) \]

\[ y = 0 \]