Math 1571H, Fall 2005
Solution to Quiz 8 (December 8)

Question 1. [5 points] Show that the work done in stretching a spring of natural length L from a length a to a length b, where $(L < a < b)$, is equal to the amount of stretch $(b-a)$ times the tension in the spring when its length is $\frac{1}{2}(a+b)$.
Solution: Let the spring constant be k. Then

$$W = \int_{a-L}^{b-L} kx dx = k\left[\frac{1}{2}\cdot x^2\right]_{a-L}^{b-L} = \frac{k}{2}((b-L)^2-(a-L)^2) = \frac{k}{2}(b-a)(a+b-2L) = (b-a)\cdot k\left(\frac{a+b}{2} - L\right).$$

Therefore, $W = (b-a) \times$ (Tension when length is $\frac{1}{2}(a+b)$).

Question 2. [5 points] Cesium 137 is used in medical and industrial radiology. Estimate its half-life if 20 percent decays in 10 years.
Solution: Let $I(t)$ be the amount of Cesium at time t. Then,

$$I(t) = I_0 e^{-kt}.$$

Therefore,

$$-k\cdot 10 = \ln(I(10)/I_0) = \ln(0.8) \Rightarrow k = \frac{\ln(1.25)}{10}\mathrm{hr}^{-1}.$$

We need to find t when $I(t) = 0.5I_0$, call it $t_{1/2}$

$$t_{1/2} = 10\frac{\ln(2)}{\ln(1.25)} \approx 31.063 \text{ hours}.$$

Question 3. [2 points Bonus!] Evaluate

$$\lim_{h\to 0}\frac{1}{h}\int_2^{2+h}\sqrt{1+t^3}dt.$$

Solution: Let $F(x) = \int_2^{2+x}\sqrt{1+t^3}dt$. Therefore from FTC, $\frac{dF}{dx} = \sqrt{1+(2+x)^3}$, and $F(0) = \int_2^2\sqrt{1+t^3} = 0$. Then,

$$\lim_{h\to 0}\frac{1}{h}\int_2^{2+h}\sqrt{1+t^3}dt = \lim_{h\to 0}\frac{F(h)-F(0)}{h} = \frac{dF(0)}{dx} = \sqrt{1+(2+0)^3} = 3.$$

Best of Luck on the Final!