Math 1571H, Fall 2005
Solution of Quiz 1 (September 15)

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1) [6 points] Sketch the graph of $y = x^2 - 4$. Find the equations of the two lines through the point $(3, 1)$ that are tangent to the curve $y = x^2 - 4$. Use the fact that if $f(x) = x^2 - 4$, then $f'(x_0) = 2x_0$.

Take a point, $(x_0, x_0^2 - 4)$, on the graph of $y = x^2 - 4$. The slope of the tangent line is $(x_0^2 - 4 - 1)/(x_0 - 3)$, which is equal to $2x_0$. Equating both, you get a quadratic equation, $x_0^2 - 5 = 2x_0^2 - 6x_0$. Solve it, you get $(x_0 - 5)(x_0 - 1) = 0$. Therefore, $x_0 = 1$, or $x_0 = 5$. Now, you have the coordinates of two points, $(1, -3)$ and $(5, 21)$. Use the equation of the line to get the equations of the two tangents.

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2) [4 points] Is $f(x) = x$ equal to $g(x) = (\sqrt{x})^2$? Justify.

NO. Since the domain of f is all real numbers, while the domain of g is all nonnegative real numbers.

[2 points] Bonus: Determine whether the following is True or False. Explain.

1) If f is a function, then $f(s + t) = f(s) + f(t)$.
FALSE. As a counter-example consider $f(x) = x^2$.

2) If f is a function and $f(s) = f(t)$, then $s = t$.
FALSE. Also, $f(x) = x^2$ can serve as a counter-example. The statement is true, if $f(x)$ is a one-to-one function.