Math 5467. Midterm Exam I (take-home problem)

March 12, 2004

This problem employs the epicycle method of Ptolemy and the theory of Copernicus: that the sun is the center of the solar system and the planets orbit the sun in circular coplanar orbits. You will use this simple approximation and truncated Fourier series to explain the apparent retrograde motion of Mars in the night sky. (Of course, to obtain more accurate orbits for the planets we would need to take into account Kepler's insight that each orbit is much better approximated by an ellipse with the sun at one of the foci, and then Newton's laws of motion that make precise computation possible.)

Earth and Mars orbit the sun with periods $T_E = 1$ year and $T_M = 1.88$ year, respectively. We consider the positions of Earth and Mars at time t in years as points $z_E(t), z_M(t)$ in the complex plane, with the sun at the origin $z_s(t) = (0,0) = \theta$. Earth is at a distance of one astronomical unit (au) from the sun, where 1 au $\sim 150 \times 10^6$ km, and Mars is 1.52 au from the sun. To simplify your verification of the retrograde motion, you can round off the period of Mars to 2 years, and its distance from the sun to 1.5 au. Thus we have (approximately)

$$z_E(t) = e^{2\pi i t}, \qquad z_M(t) = 1.5 \times e^{2\pi i t/2} = 1.5 \times e^{\pi i t},$$

where we have expressed the complex numbers z_E, z_M in polar form, and t is in years. Note that the position of Mars as seen from the Earth is $z(t) = z_M(t) - z_E(t)$, at any time t.

1. Plot the circular orbits $z_E(t), z_M(t)$ on a single graph and label the corresponding points $E(t_j) = z_E(t_j), M(t_j) = z_M(t_j)$ where $t_j = j/4$, $j = 0, 1, 2, \cdots, 8$, i.e., label the positions of Earth and Mars every 3 months over a 2 year period.

2. Plot the orbit $z(t)$ for $0 \le t \le 2$, which shows the position of Mars as seen from Earth over a 2 year period. Label the points $z(t_j)$ for $j = 0, 1, 2, \cdots, 8$. Note that this curve corresponds to an epicycle: a circle rolling on a circle.

3. For about 9 months of the year Mars traverses the night sky in the same direction as the moon. However, for about 3 months it moves in the opposite direction. Use your results from item 2 to explain the retrograde motion.