

**MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL  
EQUATIONS  
SAMPLE MIDTERM TEST II**

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You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test.

Good luck!

**Problem 1.** An object of mass 2 kg, resting on a table next to a wall, is attached to the wall by a spring. A force of 8 N is applied to the mass, stretching the spring and moving the mass  $1/2$  m from its equilibrium position. The object is then released. Suppose the resistance to the motion is numerically equal to 8 times the instantaneous velocity.

- (1) Set up an IVP governing the motion of the mass.
- (2) Determine the position of the mass at any time  $t$ .
- (3) At what time does the mass first pass through the equilibrium position and heading away from the wall?

**Problem 2.** For two matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix},$$

find  $A^T B$  and  $\text{tr}(A + 2B)$ .

**Problem 3.** Find a two by two matrix  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  such that  $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$  and none of the entries of  $A$  are zero.

**Problem 4.** Use Gauss-Jordan elimination to transform the augmented matrix of the following system into the RREF. Use it to find the solutions, if there exist any.

$$\begin{aligned} x + y - 2z &= 0, \\ 3x + 5y - 2z &= 8. \end{aligned}$$