Math 2243　　　Test 1　　　Thursday, February 6, 2003.
Professor Peter A. Rejto.

1. (15 pts.) Find solutions to the following differential equation, describing a harmonic motion:

$$\frac{d^2y(t)}{dt^2} + 4y(t) = 0.$$

2. (15 pts.) An object is released from rest at a height of $100m$ above the ground. Let $y(t)$ denote the displacement of this object from its initial position at time t. Then, neglecting frictional forces, this function $y(t)$ satisfies the initial-value problem:

$$\frac{d^2y(t)}{dt^2} = g, \;\; y(0) = 0, \;\; \frac{dy(t)}{dt}(0) = 0.$$

Here, g is a constant, the gravitational constant. Find the time when the object hits the ground. In other words, find the time t_{100} such that

$$y(t_{100}) = 100.$$

3. (20 pts.) Use "separation of variables" to find solutions to the differential equation:

$$(1+y)\frac{dy(x)}{dx} = x\cos x. \tag{1}$$

4. (a) (10 pts.)
Solve the quadratic equation,

$$y^2 + 2y - 2(x\sin x + \cos x) = 1, \tag{2}$$

for the variable y in terms of the variable x.

(b) (10 pts.) Show that your function $y = y(x)$ of part (a) satisfies the differential equation (1) of Problem 3.

5. Let the given function $y(x)$ of the variable x satisfy the algebraic equation (2) of Problem 4a. Prove that $y(x)$ also satisfies the differential equation (1) of Problem 3. Hint: Differentiate both sides of the algebraic equation (2) with respect to x and use the chain rule.

6. Find solutions to the differential equation:

$$\frac{dy}{dx} + \frac{1}{x}y = \ln x, \;\; y(1) = 3.75.$$