Math 2243, Midterm Exam 3

December 6, 2001

INSTRUCTIONS: Books and notes are not allowed. Calculators are allowed. Problems 1-3 are in "multiple choice" format. For these problems circle the answer you believe to be correct(only one of the answers listed for each problem is correct). Write *complete solutions* to problems 4-6 for full credit. You have 50 minutes to work on the problems.

Name: ______________________________ TA Section: ______________________________

1) (10 pts) Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a linear transformation with $T((1,0,1)) = (3,2,1)$, $T((0,1,0)) = (2,2,0)$, $T((0,0,1)) = (-1,-1,-1)$. Then $T((3,3,5))$ is equal to

(A) $(9,6,3)$

(B) $(13,10,1)$

(C) $(15,12,3)$

(D) $(-2,-2,1)$

(E) $(9,6,-2)$

(Hint: Try first to write $(3,3,5)$ as a linear combination of vectors whose T-values you know.)

2) (10 pts) Let $W(t)$ be the Wronskian of the vector-functions $\mathbf{x}_1(t) = \begin{bmatrix} e^{-2t} \\ \cos 3t \\ -\sin 3t \end{bmatrix}$,

$\mathbf{x}_2(t) = \begin{bmatrix} 0 \\ \sin 3t \\ \cos 3t \end{bmatrix}$, $\mathbf{x}_3(t) = \begin{bmatrix} e^{-2t} \\ -\cos 3t \\ \sin 3t \end{bmatrix}$. Then

(A) $W(0) = 2$ and the vectors are linearly independent.

(B) $W(0) = 0$ and the vectors are linearly dependent.

(C) $W(0) = 0$ and the vectors are linearly independent.

(D) $W(0) = 2$ and the vectors are linearly dependent.

(E) $W(0) = 2$ and the vectors may be either linearly dependent, or linearly independent.

3) (10 pts) Consider the differential system $\mathbf{x}' = A\mathbf{x} + \mathbf{b}(t)$, with $A = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}$. Assume you are given a solution $\mathbf{x}_p(t)$. Then the general solution has the form

(A) $c_1 e^{-t}\mathbf{v}_1 + c_2 e^{-t}\mathbf{v_2}$

(B) $c_1 e^{-t}\mathbf{v}_1 + c_2 e^{-t}\mathbf{v_2} + \mathbf{x}_p$

(C) $c_1 e^{-t}\mathbf{v}_1 + c_2 e^{-t}(\mathbf{v}_2 + t\mathbf{v}_3) + \mathbf{x}_p$

(D) $c_1 e^{t}\mathbf{v}_1 + c_2 e^{t}\mathbf{v_2} + \mathbf{x}_p$

(E) $c_1 e^{-t}(\mathbf{v}_1 + t\mathbf{v}_2) + c_2 e^{-t}(\mathbf{v}_3 + t\mathbf{v}_4) + \mathbf{x}_p$

End of multiple choice problems

4) (30 points) (a) Consider the matrix $A = \begin{bmatrix} 3 & -2 & -2 \\ 1 & 0 & -2 \\ 0 & 0 & 3 \end{bmatrix}$. Determine if A is diagonalizable. In case it is, find a matrix B such that $B^{-1}AB$ is a diagonal matrix.

(b) If A is the matrix in part (a), find the solution to the initial value problem $\mathbf{x}' = A\mathbf{x}$, $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

5) (25 pts) Find the general solution to the first-order linear differential system

$$\begin{aligned} x_1' &= 5x_1 - 5x_2 \\ x_2' &= 2x_1 - x_2 \end{aligned}$$

6) (25 pts) Find two linearly independent solutions for the first-order linear differential system $\mathbf{x}' = A\mathbf{x}$, where $A = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}$. (Note that the matrix is the same as in problem 3, so you may use any of the calculations you did for that problem - however, you need to write them down here.)