Math 1571H.   Practice Midterm Exam I

There are a total of 100 points on this exam, plus a 5 point extra credit problem that should not attempt unless you have finished the rest of the exam. To get full credit for a problem you must show the details of your work. Answers unsupported by by an argument will get little credit.

Problem 1 (15 points) *Given the curve $y = f(x) = -x^2 - 1$, for which values of x does the tangent line to the curve at (x, y) pass through the origin?*

Problem 2 (20 points) *Find the minimum distance from the origin to the plane $x + 2y + 3z = 1$.*

Problem 3 (20 points) *Let y be a function of x such that $x^2 y - y^3 = 1$ and the derivatives y', y'' exist at $x = 0$. If $y(0) = -1$, compute $y'(0)$ and $y''(0)$.*

Problem 4 (10 points) *Compute the derivative $f'(x)$ and simplify your answer. It is important that you show your work. The answer alone is not sufficient.*

$$f(x) = \left(\frac{x^2 - 1}{x^2 + 1}\right)^2$$

Problem 5 *Compute the limits. It is important that you show your work. The answer alone is not sufficient.*

a. (10 points)

$$\lim_{x \to 1} \frac{2x^2 - 3x + 1}{x^2 - x}$$

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b. (10 points)

$$\lim_{x \to 0} \frac{1 - \cos x}{\sin x \tan x}$$

Problem 6 *Consider a 4-sided pyramid whose rectangular base has vertices*

$$P(0, 0, 0), \quad Q(2, 0, 0), \quad R(0, 4, 0), \quad S(2, 4, 0)$$

and whose top vertex is $T(1, 2, 3)$.

a. (10 points) *Find the cosine of* $\angle TPS$.

b. (5 points) *Compute the area of* $\triangle TPS$.

Problem 7 (5 points extra credit) *Starting at time $t = 0$ (in seconds), a particle moves along the s-axis according to the rule $s(t) = 2t^3 - 15t^2 + 1$ where s is measured in meters. What is the furthest point to the right the particle reaches in the first 6 seconds?*

Brief answers:

1. $x = \pm 1$

2. $1/\sqrt{14}$

3. $y'(0) = 0, \quad y''(0) = -2/3$

4.
$$f'(x) = \frac{8x(x^2 - 1)}{(x^2 + 1)^3}$$

5a. 1

5b. $1/2$

6a. $12/\sqrt{154}$

6b. $\sqrt{10}$

7. $s = 1$