Name: ______________________________

Section: ______________________________

Math 1571H. Final Exam December 14, 2006

There are a total of 235 points on this exam. It is a 3 hour exam with caculators encouraged, but no notes or text. No other electronic devices such as cell phones, headphones, etc. are permitted. To get full credit for a problem you must show the details of your work. Answers unsupported by an argument will get little credit.

Problem	Score
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	
13.	
Total:	

Problem 1 *(20 points) Find the length of the curve* $y = \ln(\cos x)$ *between* $x = 0$ *and* $x = \frac{\pi}{4}$.

Problem 2 *(20 points) A cylindrical tank of diameter 4 feet is lying on its side. If the tank is half full of rum having a density of* $60\ \mathrm{lbs./ft}^3$*, what is the force on one vertical end of the tank?*

Problem 3 *Identify the graphs of the given curves expressed in polar coordinates. Be as specific as you can.*

1. *(5 points)*

$$r = 5/\sin\theta$$

2. *(5 points)*

$$r = 2/(1 - \frac{1}{2}\cos\theta)$$

Problem 4 *(20 points) Find the arc length of the curve* $\mathbf{r}(t) = t^3\ \mathbf{i} + t^2\ \mathbf{j}$ *between* $t = 1$ *and* $t = 2$.

Problem 5 *(15 points) Make the substitution* $\sin\theta = 3x$ *to evaluate the indefinite integral*

$$\int \sqrt{1-9x^2}\,dx.$$

Show the details of your work.

Problem 6 *(20 points) Find the local maxima, the local minima, and the inflection points of the function* $f(x) = e^x - 3e^{-x} - 4x$.

Problem 7 *(20 points) Solve the differential equation with initial condition*

$$\frac{dy}{dx} = \frac{2y+3}{x+5}, \qquad y(0) = 1.$$

Problem 8 *(20 points) Suppose the region under the graph of the curve*

$$y = \frac{1}{3}(x^2+2)^{\frac{3}{2}}, \qquad 1 \leq x \leq 3$$

is rotated about the y-axis. Find the volume of the solid generated.

Problem 9 *(20 points) Find the equation of the plane containing the point* $P(0, 1, -1)$ *and the line with vector equation*

$$\mathbf{R}(t) = (3 + t)\ \mathbf{i} + (1 + t)\ \mathbf{j} + (2 - t)\ \mathbf{k}.$$

Problem 10 *(10 points)*

$$F(x) = \int_{5x^2-1}^{1} \sin(t^3)\ dt.$$

Compute $F'(x)$.

Problem 11 *(20 points) Given that* $\sum_{j=1}^{n} j^3 = \frac{n^2(n+1)^2}{4}$, *use Riemann sums to compute the area of the region bounded by the curve* $y = x^3$, *the* x *axis and the line* $x = 1$.

Problem 12 *(20 points) In a laboratory there are 10 grams of a radioactive substance with a half-life of 20 years. How many grams of the substance will remain after 25 years?*

Problem 13 *(20 points) The front of a tank which is full of water has the shape of a regular trapezoid. In a suitable Cartesian coordinate system* (x, y) *the coordinates of the vertices of the top of the trapezoid are* $(-4, 10)$ *and* $(4, 10)$*, and the vertices of the bottom are* $(-12, 0)$ *and* $(12, 0)$*. Find the hydrostatic force exerted by the water on this tank front. The lengths are in feet and the water density is* 62.5 *lb/ft*^{3}.

Brief solutions:

1. $\ln(1+\sqrt{2})$
2. 320 lbs.
3. 1) vertical line $y = 5$, 2) ellipse in standard position $e = \frac{1}{2}$, $p = 4$.
4. $$\frac{1}{27}(40\sqrt{40} - 13\sqrt{13})$$
5. $$\frac{1}{6}\arcsin(3x) + \frac{x}{2}\sqrt{1-9x^2} + c$$
6. local max. at $x = 0$, local min. at $x = \ln 3$, inflection pt. at $x = \frac{1}{2}\ln 3$
7. $$y = \frac{1}{10}(x+5)^2 - \frac{3}{2}$$
8. $$\frac{2\pi}{15}(11^{\frac{5}{2}} - 3^{\frac{5}{2}})$$
9. $$x - 2y - z = -1$$
10. $$-10x\sin[(5x^2-1)^3]$$
11. 1/4
12. $5 \times 2^{-1/4} \approx 4.204$ grams
13. $$\frac{35}{6}10^4 \text{ lbs.}$$