

Name: _____

Section: _____

Math 1571H. Practice Midterm Exam III November 29, 2006

There are a total of 100 points on this exam, plus one 5-point extra credit problem that you should only work if you complete the rest of the exam. To get full credit for a problem you must show the details of your work. Answers unsupported by an argument will get little credit.

Problem	Score
1.	_____
2.	_____
3.	_____
4.	_____
5.	_____
6.	_____
Extra credit	_____
Total:	_____

Problem 1 (15 points) Find the derivative $f'(x)$ and simplify.

1. $f(x) = \ln(\sec x + \tan x)$

2. $f(x) = \int_{\sin x}^6 (\cos t)^t dt$

3. $f(x) = (\sin x)^{2 \tan x}$ Don't simplify in this case.

Problem 2 (15 points) Find the volume swept out when the area under the top half of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is rotated about the x -axis.

Problem 3 (15 points) Find the volume swept out when the area under the curve

$$y = \sin(x^2), \quad 0 \leq x \leq \sqrt{\pi}$$

is rotated about the y -axis.

Problem 4 (20 points) Find the length of the curve with vector equation

$$\mathbf{R}(t) = e^t \sin t \mathbf{i} + e^t \cos t \mathbf{j}$$

between $t_0 = 0$ and $t_1 = \frac{1}{2} \ln 2$.

Problem 5 (15 points) *A spring has a natural length of 15 in. and a 10 lb. weight stretches it 2 in. How much work is done in stretching the spring from -2 in. to +3 in.?*

Problem 6 (20 points) *A dam has a vertical side in the shape of a right triangle with vertex at the bottom, height of 10 ft. and width of 12 ft. at the top. The water (density 62.5 lbs./ft.³) behind the dam is 8 feet deep. Compute the total force of the water against the dam.*

Problem 7 (*EXTRA CREDIT, 5 points*) *The function*

$$f(x) = |(x + 2)(x - 1)|$$

is continuous everywhere, so it has an antiderivative. Compute the antiderivative $F(x)$ on the domain $x > 0$ such that $F(1) = 0$. Simplify your answer!

Brief solutions.

1. 1) $\sec x$, 2) $-(\cos[\sin x])^{\sin x} \cos x$, 3)

$$(\sin x)^{2 \tan x} [2 \sec^2 x \ln(\sin x) + 2]$$

2. $\frac{4\pi ab^2}{3}$

3. 2π

4. $2 - \sqrt{2}$

5. 12.5 in-lbs.

6. 6400 lbs.

7.

$$f(x) = \begin{cases} \frac{1}{3}(x-1)^3 + \frac{3}{2}(x-1)^2, & x \geq 1 \\ -\frac{1}{3}(x-1)^3 - \frac{3}{2}(x-1)^2, & 0 < x < 1 \end{cases}$$