Name: ______________________________

Section: ______________________________

Math 1571H. Practice Midterm Exam II November 1, 2006

There are a total of 100 points on this exam, plus one 5-point extra credit problem that you should only work if you complete the rest of the exam. To get full credit for a problem you must show the details of your work. Answers unsupported by by an argument will get little credit.

Problem	Score
1.	______
2.	______
3.	______
4.	______
5.	______
6.	______
Extra credit	______
Total:	______

Problem 1 *(20 points) Find the dimensions of the rectangle of maximum area that can be inscribed in an equilateral triangle with sides of length a, with one side of the rectangle lying on one side of the triangle.*

Problem 2 *(15 points) Find the solution $y(x)$ of the differential equation*

$$\frac{dy}{dx} = \frac{\cos x}{y}$$

such that $y = \sqrt{2}$ when $x = \pi/2$.

Problem 3 *Compute the integrals. Note that some of these integrals are indefinite and some definite.*

a. (5 points)

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \sec^2(2x)\ dx$$

b. (5 points)

$$\int_1^3 \frac{4x^2\ dx}{(1+x^3)^{\frac{2}{3}}}$$

c. (5 points)

$$\int \frac{3x^{\frac{2}{3}} + 6x^{\frac{1}{2}} - x}{x^{\frac{1}{3}}}\ dx$$

d. (5 points)

$$\int \frac{\cos x \sin x}{(3 + \cos^2 x)^2} \; dx$$

Problem 4 *(15 points)* *After passing over a ground station an airplane flies in a horizontal line at an altitude of 3 miles. At a certain instant, it is found that the distance from the airplane to the ground station is 5 miles, and that this distance is increasing at the rate of 500 mph. How fast is the airplane flying?*

Problem 5 *The vector equation of the position of a particle in the plane is given by the epicycle*

$$\mathbf{R}(t) = (2\cos t + \cos 2t)\mathbf{i} + (2\sin t + \sin 2t)\mathbf{j}$$

where t is the time. The double angle formulas

$$\sin 2t = 2\sin t\cos t, \qquad \cos 2t = 2\cos^2 t - 1 = 1 - 2\sin^2 t$$

may be relevant to this problem.

a. (10 points) *Is there a time (or times) t at which the velocity of the particle is $\mathbf{0}$, so that the particle is momentarily at rest? If so find the times.*

b. (5 points) *Find the vector equation of the tangent line to this curve at the time $t = \frac{\pi}{2}$.*

Problem 6 *(15 points) Use differentials to find an approximation to* $(26.96)^{\frac{1}{3}}$.

Problem 7 *(EXTRA CREDIT, 5 points) There is a root of the polynomial* $p(x) = x^3 + x - 1$ *in the interval* $[0, 1]$. *We make an initial guess* $x_1 = \frac{1}{2}$ *for this root. Use Newton's method to compute the next approximation* x_2.

Solutions:

1. Height $\frac{a\sqrt{3}}{4}$, Width $\frac{a}{2}$

2. $y = \sqrt{2 \sin x}$, in domain $0 < x < \pi$

3a. $\frac{1}{2}(1 - \frac{\sqrt{3}}{3})$

3b. $4(28^{\frac{1}{3}} - 2^{\frac{1}{3}})$

3c. $\frac{9}{4}x^{\frac{4}{3}} + \frac{36}{7}x^{\frac{7}{6}} - \frac{3}{5}x^{\frac{5}{3}} + C$

3d. $\frac{1}{2(3+\cos^2 x)} + C$

4. 625 m.p.h.

5a. $t = \pi + 2\pi k, \qquad k = 0, \pm 1, \pm 2, \cdots$

5b. $\mathbf{r}(s) = (-1 - s)\mathbf{i} + (2 - s)\mathbf{j}$

6. $(26.96)^{\frac{1}{3}} \sim \frac{2024}{675} \sim 2.998519$ Actually, $(26.96)^{\frac{1}{3}} \sim 2.998518$

7. $x_2 = \frac{5}{7}$