

Name: \_\_\_\_\_

Signature: \_\_\_\_\_

Section: \_\_\_\_\_

**Math 1271. Lecture 030 Practice Midterm Exam II**

There are a total of 100 points on this exam. To get full credit for a problem you must show the details of your work. Answers unsupported by an argument will get little credit. No books, notes, calculators, cell phones or other electronic devices are allowed. Do all of your calculations on this test paper.

Problem	Score
1.	_____
2.	_____
3.	_____
4.	_____
5.	_____
Total:	_____

**Problem 1** Compute the indicated derivatives of the functions  $y = f(x)$ . It is not necessary to simplify.

**a. (5 points)**  $f'(x)$  where

$$f(x) = \frac{\sin(1 - e^x)}{x}.$$

**b.(5 points)**  $y'$  where

$$y = (x + 3)^{10}(x^2 - 9)^6(x - 1)^7.$$

**c.(5 points)**  $f'(x)$  where

$$f(x) = x \arcsin(x).$$

**d.(5 points)**  $f^{(7)}(x)$  where

$$f(x) = \frac{1}{2x+1}.$$

**Problem 2** (20 points) Sand falling at the rate of  $3 \text{ ft}^3/\text{min}$  forms a conical pile whose radius  $r$  always equals twice the height  $h$ . Find the rate at which the height is changing at the instant when the height is 10 feet. Recall that the volume  $V$  of a right circular cone is  $V = \frac{1}{3}\pi r^2 h$ .

**Problem 3** Let  $y$  be a function of  $x$  such that  $x^2y - y^3 = 1$  and the derivatives  $y'$  and  $y''$  exist at  $x = 0$ .

**a. (15 points)** If  $y(0) = -1$ , compute  $y'(0)$ .

**b. (5 points)** Also compute  $y''(0)$ .

**Problem 4** (20 points) Find the absolute maximum and the absolute minimum of the function

$$f(x) = \frac{x^2 + 20}{2x + 1}$$

on the interval  $0 \leq x \leq 8$ .

**Problem 5** Let

$$y = f(x) = 2x^4 - 9x^2 + 5$$

with natural domain.

**a. (10 points)** Determine where  $f(x)$  is increasing and where it is decreasing. Find all local extrema  $(x, y)$  and identify whether they are local maxima or minima.

**b. (5 points)** Determine where  $f(x)$  is concave up and where it is concave down. Find all inflection points.

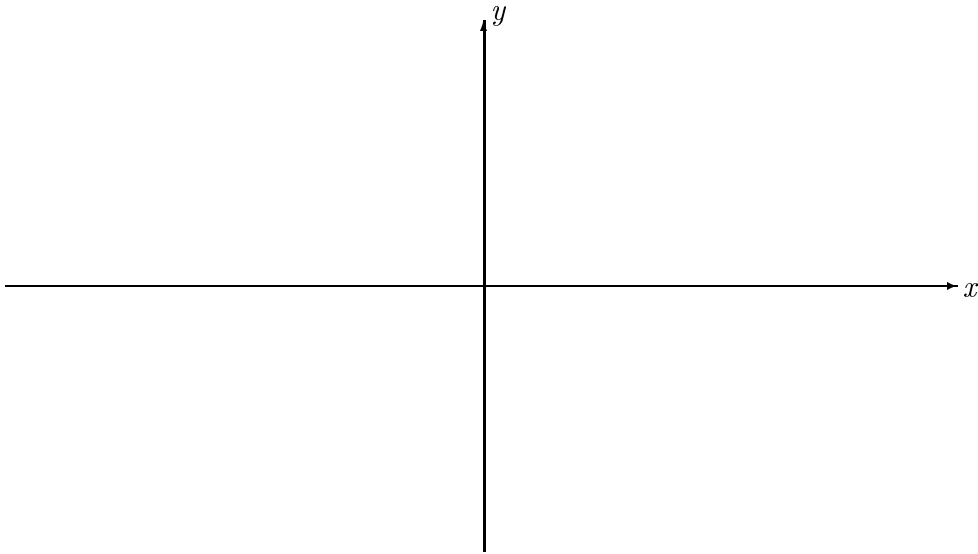


Figure 1: Graph of the curve  $y = 2x^4 - 9x^2 + 5$

- c. (5 points) *Sketch the curve, pointing out significant features such as intercepts, intervals of increase and decrease, relative extrema, inflection points, concavity and behavior approaching infinity.*

(Very ) Brief Solutions:

1a.

$$f'(x) = \frac{-xe^x \cos(1 - e^x) - \sin(1 - e^x)}{x^2}$$

1b.

$$y' = (x + 3)^{10}(x^2 - 9)^6(x - 1)^7 \left[ \frac{10}{x + 3} + \frac{12x}{x^2 - 9} + \frac{7}{x - 1} \right]$$

1c.

$$f'(x) = \arcsin(x) + \frac{x}{\sqrt{1 - x^2}}$$

1d.

$$f^{(7)}(x) = -\frac{2^7(7!)}{(2x + 1)^8}$$

2

$$\frac{dh}{dt} \Big|_{h=10} = \frac{3}{400\pi} \text{ ft./min.}$$

3a.

$$y'(0) = 0$$

3b.

$$y''(0) = -2/3$$

4. Absolute max. is 20, at point  $(x, y) = (0, 20)$ . Absolute min. is 4, at point  $(x, y) = (4, 4)$ .

5a. Increasing on intervals  $(-3/2, 0)$  and  $(3/2, \infty)$ . Decreasing on intervals  $(-\infty, -3/2)$  and  $(0, 3/2)$ . Relative and absolute minima at points  $(x, y) = (-3/2, -41/8)$  and  $(x, y) = (3/2, -41/8)$ . Relative maximum at  $(x, y) = (0, 5)$ .

5b. Concave up on intervals  $(\infty, -\sqrt{3}/2)$  and  $(\sqrt{3}/2, \infty)$ . Concave down on interval  $(-\sqrt{3}/2, \sqrt{3}/2)$ . Inflection points at  $(x, y) = (\pm\sqrt{3}/2, -5/8)$ .

5c.  $x$ -intercepts at  $x = \pm\sqrt{\frac{9+\sqrt{41}}{4}}$  or  $x = \pm\sqrt{\frac{9-\sqrt{41}}{4}}$ .  $y$ -intercept at  $y = 5$ .  
 $y \rightarrow \infty$  as  $x \rightarrow \pm\infty$ .